

EE359 – Lecture 8 Outline

● Announcements

- Makeup class tomorrow, 10/20 (no lecture next Tues 10/24) at 10:30AM in Thornton 102 (here, with donuts)
 - Tom's OHs 10/20: 9:30-10:20 (3rd Floor Packard), email OHs 1-2pm
- New discussion section and OH times starting next week
 - Wednesdays 5-6pm Discussion session (Packard 361), Tom's OHs afterwards
- Project proposals due 10/28; I can provide early feedback
- **Midterm Nov. 9, 6-8pm** (pizza after), more details next week
 - Email me/TAs if you have a conflict

● Capacity of Fading channels

- Recap Optimal Rate/Power Adaptation with TX/RX CSI
- Channel Inversion with Fixed Rate

● Capacity of Freq. Selective Fading Channels

● Linear Digital Modulation Review

● Performance of Linear Modulation in AWGN

Review of Last Lecture

- **Channel Capacity**
 - Maximum data rate that can be transmitted over a channel with arbitrarily small error
- **Capacity of AWGN Channel: $B \log_2[1+\gamma]$ bps**
 - $\gamma = P_r / (N_0 B)$ is the receiver SNR
- **Capacity of Flat-Fading Channels**
 - Nothing known: capacity typically zero
 - Fading Statistics Known (few results)
 - Fading Known at RX (average capacity)

$$C = \int_0^{\infty} B \log_2(1 + \gamma) p(\gamma) d\gamma \leq B \log_2(1 + \bar{\gamma})$$

Review of Last Lecture (ctd)

- Capacity in Flat-Fading: γ known at TX/RX

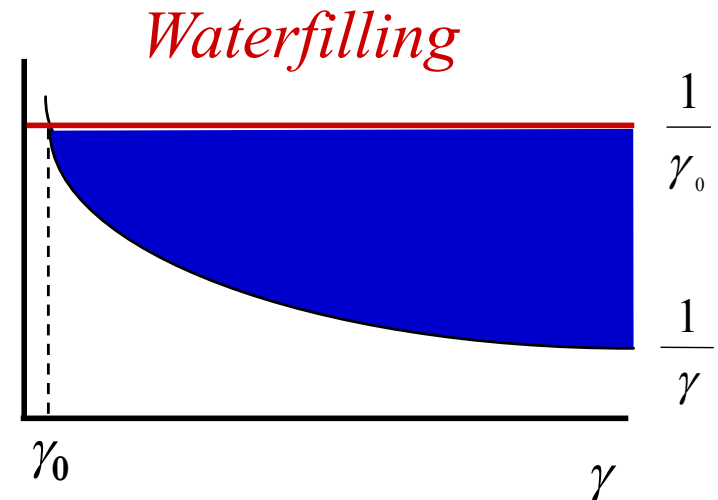
$$C = \max_{P(\gamma) : E[P(\gamma)] \leq \bar{P}} \int_0^{\infty} B \log_2 \left(1 + \frac{\gamma P(\gamma)}{\bar{P}} \right) p(\gamma) d\gamma$$

Same result with equality

- Optimal Rate and Power Adaptation

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \text{else} \end{cases}$$

$$\frac{C}{B} = \int_{\gamma_0}^{\infty} \log_2 \left(\frac{\gamma}{\gamma_0} \right) p(\gamma) d\gamma.$$



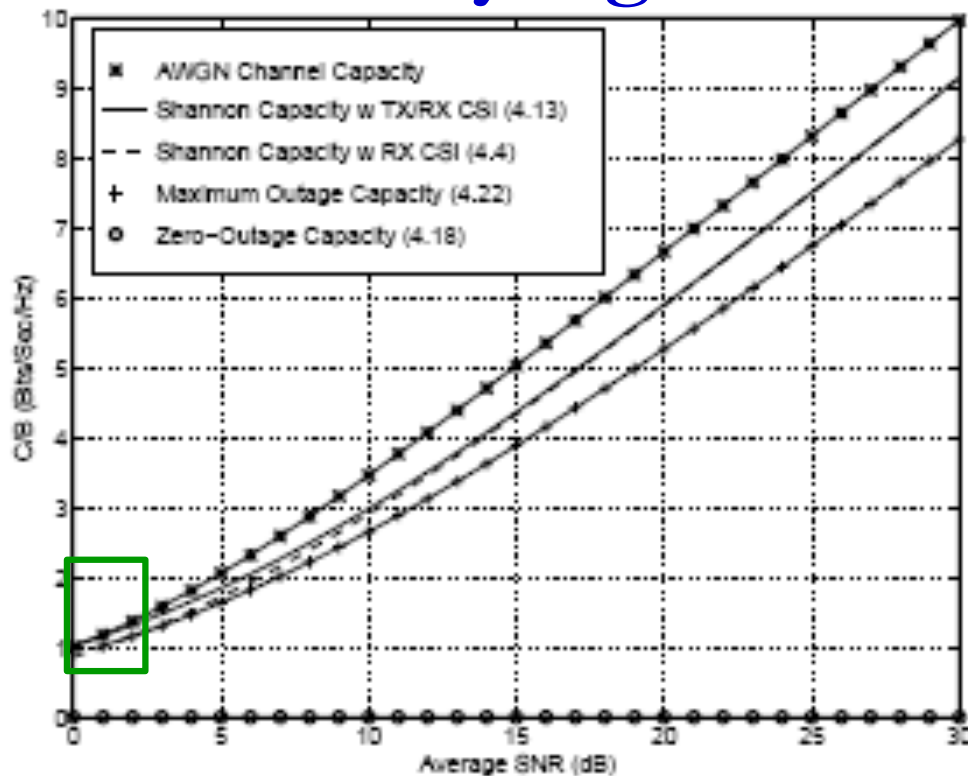
- The instantaneous power/rate only depend on $p(\gamma)$ through γ_0

Channel Inversion

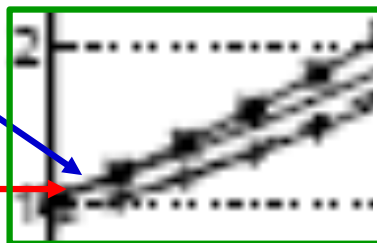
- Fading inverted to maintain constant SNR
- Simplifies design (fixed rate)
- Greatly reduces capacity
 - Capacity is zero in Rayleigh fading
- Truncated inversion
 - Invert channel above cutoff fade depth
 - Constant SNR (fixed rate) above cutoff
 - Cutoff greatly increases capacity
 - Close to optimal

Capacity in Flat-Fading

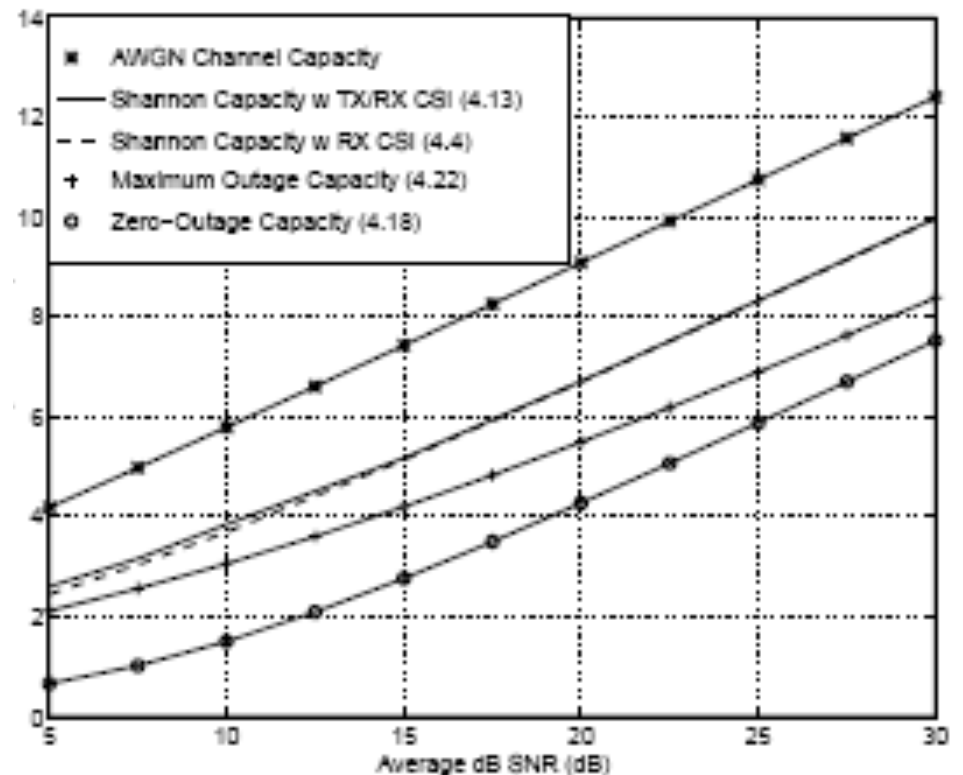
Rayleigh



AWGN capacity
lower than w/fading
under TX/RX CSI

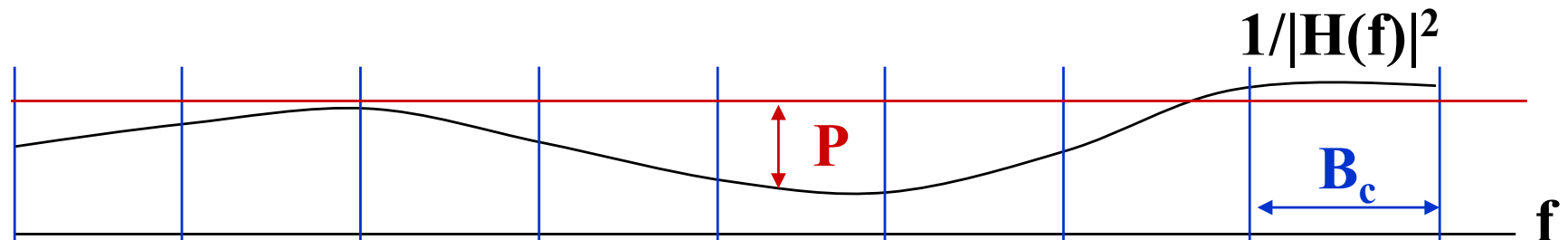


Log-Normal



Frequency Selective Fading Channels

- For time-invariant channels, capacity achieved by water-filling in frequency
- Capacity of time-varying channel unknown
- Approximate by dividing into subbands
 - Each subband has width B_c (like MCM/OFDM).
 - Independent fading in each subband
 - Capacity is the sum of subband capacities



Review of Linear Digital Modulation

- Signal over i th symbol period:

$$s(t) = s_{i1}g(t)\cos(2\pi f_c t + \phi_0) - s_{i2}g(t)\sin(2\pi f_c t + \phi_0)$$

- Pulse shape $g(t)$ typically Nyquist
 - Signal constellation defined by (s_{i1}, s_{i2}) pairs
 - Can be differentially encoded
 - M values for $(s_{i1}, s_{i2}) \Rightarrow \log_2 M$ bits per symbol
- P_s depends on
 - Minimum distance d_{min} (*depends on γ_s*)
 - # of nearest neighbors α_M
 - Approximate expression:
 - Standard/alternate Q function

$$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$$

Main Points

- Channel inversion practical, but should truncate or get a large capacity loss
- Capacity of wideband channel obtained by breaking up channel into subbands
 - Similar to multicarrier modulation
- Linear modulation dominant in high-rate wireless systems due to its spectral efficiency
- Ps approximation in AWGN: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
 - Alternate Q function useful in diversity analysis