EE359 – Lecture 4 Outline

- Announcements:
  - Next week OHs: Milind's OHs are W 5-6pm, Th 4-5pm and email OHs 5-6pm. Tom's OHs are Friday (10/13) 1-4pm (in person/email)
  - 1st HW due tomorrow 4pm: HW can be submitted in hardcopy (box outside Julia's office) or upload to canvas (see website).
- Review of Last Lecture
- Random Multipath Model
- Time Varying Channel Impulse Response
- Narrowband Fading Model
- In-Phase and Quad Signal Components
- Auto and Crosscorrelation of received signal

Review of Last Lecture

- **Shadowing: Log-normal random variable based on CLT applied to many attenuating objects**

- **Combined Path Loss and Shadowing**

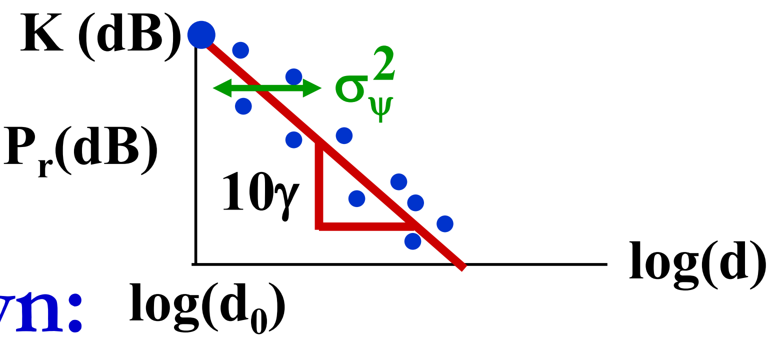
$$\frac{P_r}{P_t}(dB) = \underbrace{10\log_{10} K}_{K_{dB}} - 10\gamma \log_{10}\left(\frac{d}{d_0}\right) - \psi_{dB}, \quad \psi_{dB} \sim N(\mu_\psi, \sigma_\psi^2)$$

μ_ψ=0 when average shadowing incorporated into K and γ, else μ_ψ>0

- **Outage probability:** $p_{out}(P_{min}, d) = p(P_r(d) < P_{min})$
- **For log-normal shadowing model**

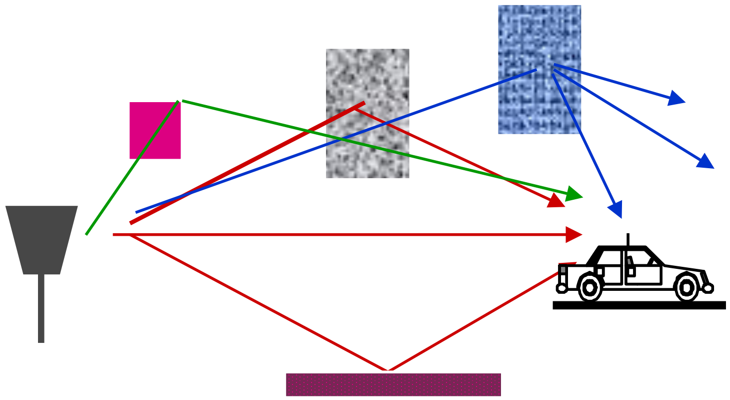
$$p(P_r(d) \le P_{min}) = 1 - Q\left(\frac{P_{min} - (\overbrace{P_t + K_{\mathrm{dB}} - 10\gamma \log_{10}(d/d_0)}^{E[P_r(d)]})}{\sigma_{\psi_{dB}}}\right)$$

Review continued



- Fit model to data
- Path loss (K,γ), d_0 known:
  - "Best fit" line through dB data:
  - 1D (γ) or 2D (γ,K) minimization of the MSE
  - For 1D, K obtained from free-space or at d_0.
  - Captures mean due to shadowing
- Shadowing variance
  - Variance of data relative to path loss model (straight line) with MMSE estimate for γ

Statistical Multipath Model



- Random # of multipath components, each with
  - Random amplitude
  - Random phase
  - Random Doppler shift
  - Random delay
- Random components change with time
- Leads to time-varying channel impulse response

Time Varying Impulse Response

- Response of channel at t to impulse at t-τ:

$$c(\tau,t)=\sum_{n=0}^{N(t)}\alpha_n(t)e^{-j\varphi_n(t)}\delta(\tau-\tau_n(t))$$

  - t is time when impulse response is observed
  - t-τ is time when impulse put into the channel
  - τ is how long ago impulse was put into the channel for the current observation
    - path delay for MP component currently observed

Received Signal Characteristics

- Received signal consists of many multipath components
- Amplitudes change slowly
- Phases change rapidly
  - Constructive and destructive addition of signal components
  - Amplitude fading of received signal (both wideband and narrowband signals)

Narrowband Model

- Assume delay spread $\max_{m,n}|\tau_n(t)-\tau_m(t)| << 1/B$
- Then $u(t) \approx u(t-\tau)$.
- Received signal given by

$$r(t) = \Re\left\{u(t)e^{j2\pi f_c t}\left[\sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)}\right]\right\}$$

- No signal distortion (spreading in time)
- Multipath affects complex scale factor in brackets.
- Characterize scale factor by setting $u(t)=e^{j\phi_0}$:

$$s(t) = \Re\{e^{j(2\pi f_c t+\phi_0)}\} = \cos(2\pi f_c t - \phi_0)$$

In-Phase and Quadrature under CLT Approximation

- In phase and quadrature signal components:

$$r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \cos(2\pi f_c t),$$

$$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0$$

$$r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t) e^{-j\phi_n(t)} \sin(2\pi f_c t)$$

- For *N(t)* large, $r_I(t)$ and $r_Q(t)$ jointly Gaussian by CLT (sum of large # of random vars).
- Received signal characterized by its mean, autocorrelation, and cross correlation.
- If $\varphi_n(t)$ uniform, the in-phase/quad components are mean zero, independent, and stationary.

Auto and Cross Correlation

- Assume $\phi_n \sim U[0,2\pi]$
- Recall that θ_n is the multipath arrival angle
- Autocorrelation of inphase/quad signal is

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = PE_{\theta_n}[\cos 2\pi f_{D_n}\tau], \quad f_{D_n} = v\cos\theta_n / \lambda$$

- Cross Correlation of inphase/quad signal is

$$A_{r_I,r_Q}(\tau) = PE_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

- Autocorrelation of received signal is

$$A_r(\tau) = A_{r_I}(\tau)\cos(2\pi f_c\tau) - A_{r_I,r_Q}(\tau)\sin(2\pi f_c\tau)$$

Main Points

- Statistical multipath model leads to a time-varying channel impulse response
- Resulting received signal has rapidly varying amplitude due to constructive and destructive multipath combining
- Narrowband model has in-phase and quad. comps that are zero-mean stationary Gaussian processes
  - Auto and cross correlation depends on AOAs of multipath