

# EE359 – Lecture 17 Outline

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- **Announcements**

- Thu lecture move to Fri, 10:30-11:50, here; Tom's Fri OH 9:30-10:30
- HW due Friday
- Last HW will be posted Thurs, due Fri of dead week (no late HWs)
- Last lecture 12/7 will be 10:30-11:30 (course review) and 11:30-12:30 (advanced topics; bonus lecture).

- **Multicarrier Modulation**

- **Overlapping subcarriers in MCM**

- **FFT implementation of MCM (OFDM)**

- **Implementation Challenges in OFDM**

- **Fading across Subcarriers**

- **MIMO-OFDM**

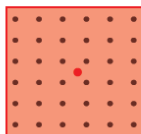
# Review of Last Lecture

- MIMO RX Design (see supplemental handout):
  - Optimal Receiver is ML: finds input symbol most likely to have resulted in received vector, exponentially complex in  $M_t$
  - Linear Receivers: First performs linear equalization:  $\tilde{x} = Ay$   
 then quantizes  $\tilde{x}$  to nearest constellation point  $x \in \mathcal{X}^{M_t}$ 
    - Zero-Forcing ( $A = \mathbf{H}^\dagger$ , the Moore-Penrose pseudo inverse of  $\mathbf{H}$ ): (if  $\mathbf{H}$  invertible, equals inverse, else  $\mathbf{H}^\dagger = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ ); forces off-diagonal terms to zero ( $\tilde{x}_i = x_i + \tilde{n}_i$ ;  $\tilde{n} = \mathbf{H}^\dagger n$ , enhances noise)
    - Minimum Mean Square Error ( $A = \mathbf{H}^H (\mathbf{H} \mathbf{H}^H + \lambda \mathbf{I})^{-1}$ ):  $\lambda \propto 1/\text{SNR}$   
Balances zero forcing against noise enhancement
- Sphere Decoder: Uses QR decomposition of  $\mathbf{H}$ 
  - Considers possibilities within sphere of transformed received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

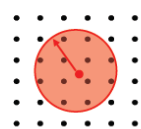
$$\hat{x} = \arg \min_x |y - Hx|^2$$

$Hx+n$

ML Decoding



Sphere Decoding

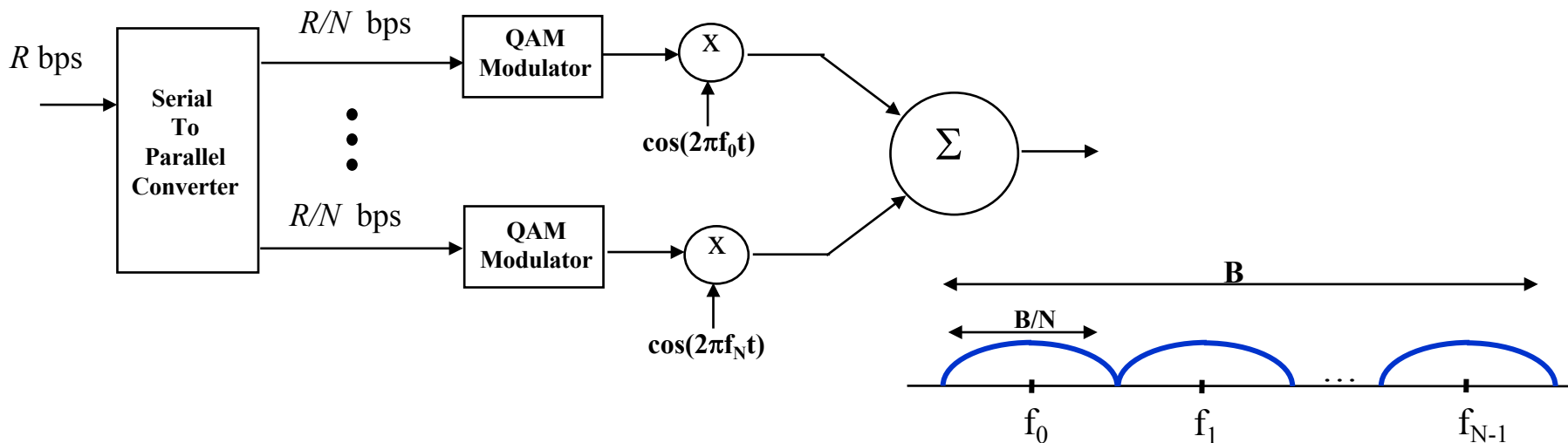


$$\hat{x} = \arg \min_{x: |Q^H y - Rx| < r} |Q^H y - Rx|^2$$

$Q^H y = Rx + Q^H n$

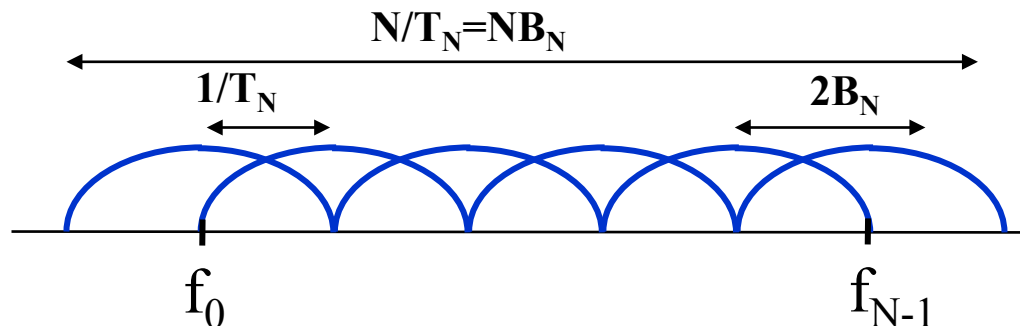
# Multicarrier Modulation

- Can mitigate ISI with equalization (not commonly used or covered), multicarrier modulation, or spread spectrum
- Multicarrier Modulation: breaks data into  $N$  substreams ( $B/N < B_c$ ); Substreams modulated onto separate carriers
  - Substream passband BW is  $B/N$  for  $B$  total BW
  - $B/N < B_c$  implies flat fading on each subcarrier (no ISI)



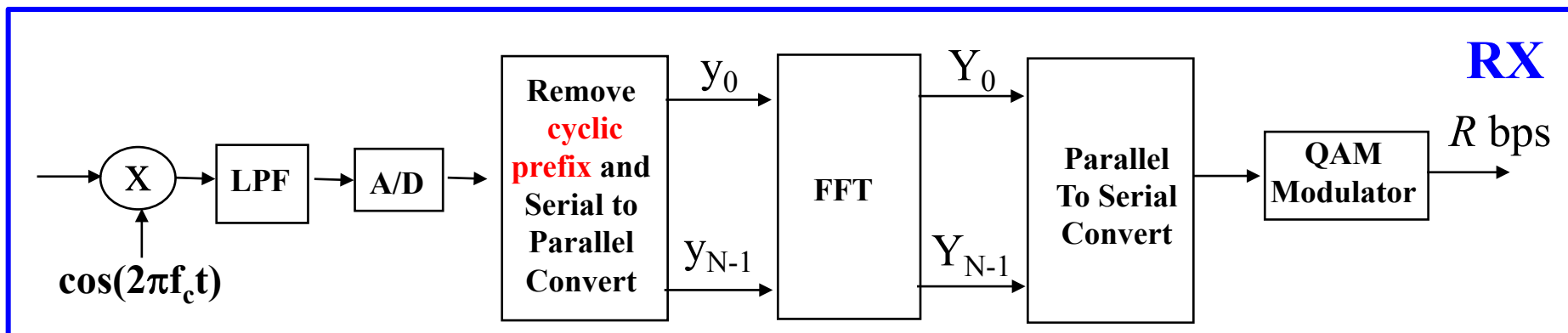
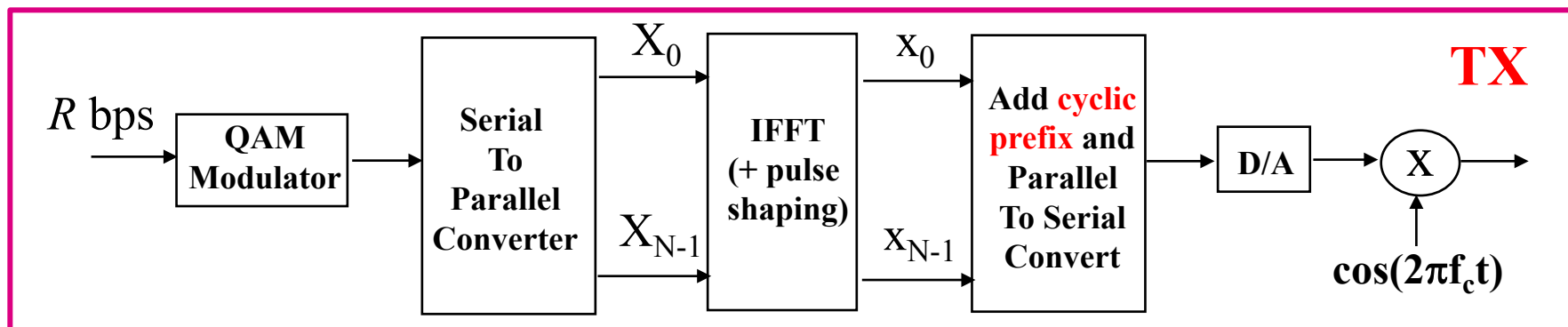
# Overlapping Substreams

- Can have completely separate subchannels
  - Required passband bandwidth is  $B$ .
- MCM with overlapping substreams
  - Substreams (symbol time  $T_N$ ) separated in RX
  - Minimum substream separation is  $1/T_N$  for rectangular pulses
  - Total required bandwidth is  $B/2$



# FFT Implementation of MCM (OFDM)

- Use IFFT at TX to modulate symbols on each subcarrier
- Cyclic prefix makes linear convolution of channel circular, so no interference between FFT blocks in RX processing
- Reverse structure (with FFT) at receiver



# OFDM Design Issues

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- **Timing/frequency offset:**
  - Impacts subcarrier orthogonality; self-interference
- **Peak-to-Average Power Ratio (PAPR)**
  - Adding subcarrier signals creates large signal peaks
  - Solve with clipping or PAPR-optimized coding
- **Different fading across subcarriers**
  - Mitigate by precoding (fading inversion), adaptive modulation over frequency, and coding across subcarriers
- **MIMO-OFDM**
  - Apply OFDM across each spatial dimension
  - Can adapt across space, time, and frequency
  - MIMO-OFDM represented by a matrix, extends matrix representation of OFDM alone (considered in HW)

# Main Points

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- MCM splits channel into NB flat fading subchannels
  - Overlapping subcarriers in OFDM reduces BW by 2x
- MCM implemented with IFFTs/FFT (OFDM)
  - Block size depends on data rate relative to delay spread
- OFDM challenges: timing/frequency offset, PAPR
- Subcarrier fading degrades OFDM performance
  - Compensate through precoding (channel inversion), coding across subcarriers, or adaptation
- OFDM naturally combined with MIMO
  - Orthogonal in space/freq; extended matrix representation
  - 4G Cellular and 802.11n/ac/ax all use OFDM+MIMO