MIMO Communications and Capacity.

Lecture Outline

- **MIMO Systems**
- **MIMO Channel Decomposition**
- **MIMO Channel Capacity**
- **Massive MIMO Channel Capacity**
- **Beamforming**

1. **MIMO Systems**

   - MIMO systems have multiple antennas at the transmitter and receiver.
   - The antennas can be used for capacity gain and/or diversity gain.
   - MIMO system design and analysis can be complex since it requires vector signal processing.
   - The performance and complexity of MIMO systems depends on what is known about the channel at both the transmitter and receiver

2. **MIMO Channel Decomposition**

   - With perfect channel estimates at the transmitter and receiver, the MIMO channel decomposes into $R_{\mathbf{H}}$ independent parallel channels, where $R_{\mathbf{H}}$ is the rank of the channel matrix ($\min(M_t, M_r)$ for M_t transmit and M_r receive antennas under rich scattering).
   - With this decomposition there is no need for vector signal processing.
   - Decomposition is obtained by transmit precoding and receiver shaping.

3. **MIMO Channel Capacity: Static Channels**

   - Capacity depends on whether the channel is static or fading, and what is known about the channel at the transmitter and receiver.
   - For a static channel known at the transmitter and receiver capacity is given by

   $$C = \max_{P_i:\sum_i P_i \le P} \sum_i B \log_2\left(1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right) = \max_{P_i:\sum_i P_i \le P} \sum_i B \log_2\left(1 + \frac{P_i \gamma_i}{P}\right).$$

   This leads to a water-filling power allocation in space.
   - Without transmitter knowledge, outage probability is the right metric for capacity.
   - In the limit of a large antenna array (Massive MIMO), even without TX CSI, random matrix theory dictates that the singular values of the channel matrix converge to the same constant. Hence, the capacity of each spatial dimension is the same, and the total system capacity is $C = \min(M_t, M_r) B \log(1+\rho)$. So capacity grows linearly with the size of the antenna arrays in Massive MIMO systems.

4. **MIMO Channel Capacity: Fading Channels**

   - In fading, if the channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate to transmit at:

$$P_{out} = p\left(\mathbf{H} : B \log_2 \det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] > C\right).$$

   - Capacity with both transmitter and receiver knowledge of the fading is the average of the capacity for the static channel, with power allocated either by an instantaneous or average power constraint. Under the instantaneous constraint power is optimally allocated over the spatial dimension only. Under the average constraint it is allocated over both space and time.

5. **Beamforming**

   - Beamforming sends the same symbol over each transmit antenna with a different scale factor.
   - At the receiver, all received signals are coherently combined using a different scale factor.
   - This produces a transmit/receiver diversity system, whose SNR can be maximized by optimizing the scale factors (MRC).
   - Beamforming leads to a much higher SNR than on the individual channels in the parallel channel decomposition.
   - Thus, there is a design tradeoff in MIMO systems between capacity and diversity.

Main Points

- MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain.
- With both TX and RX CSI, multiple antennas at both transmitter and receiver lead to independent parallel channels.
- With TX and RX CSI, static channel capacity is the sum of capacity on each spatial dimension.
- Without TX CSI, use outage as capacity metric.
- For large arrays, random gains become static, and capacity increases linearly with the number of TX/RX antennas.
- With TX and RX CSI, capacity of MIMO fading channel uses waterfilling in space or space/time - leads to $\min(M_t, M_r)$ capacity gain.
- Beamforming transforms MIMO system into a SISO system with TX and RX diversity. Beamform along direction of maximum singular value