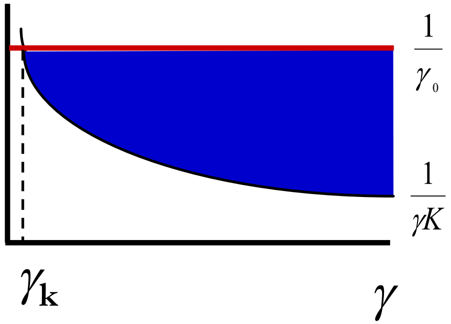
EE359 – Lecture 14 Outline

- Announcements:
  - MT tonight, 6-8pm here
  - HW posted today, due next Friday
- Introduction to MIMO Communications
- MIMO Channel Decomposition
- MIMO Channel Capacity
- MIMO Beamforming

Review of Last Lecture

- Introduction to adaptive modulation
  - Vary different parameters of modulation relative to fading
- Variable-rate variable-power MQAM
  - Maximize average throughput by changing rate and power
  - Optimal power adaptation is water-filling

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma K} & \gamma \geq \frac{\gamma_0}{K} = \gamma_K \\ 0 & \text{else} \end{cases}$$



  - Optimal rate adaptation:

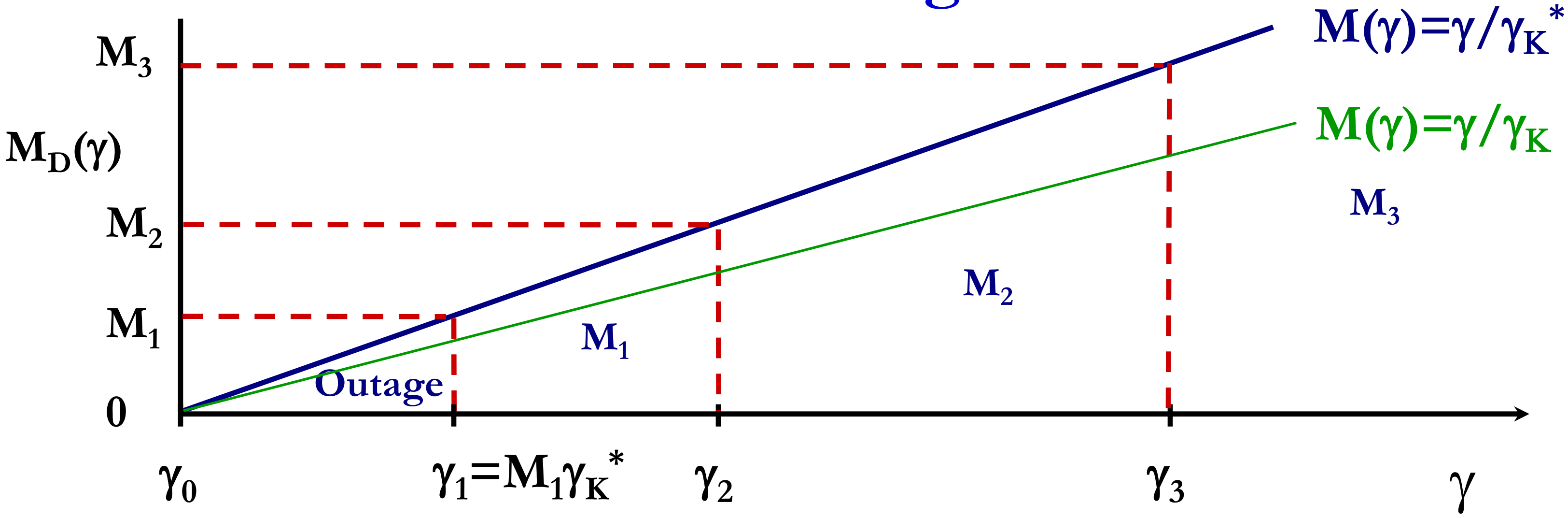
$$\frac{R}{B} = \int_{\gamma_K}^{\infty} \log_2\left(\frac{\gamma}{\gamma_K}\right) p(\gamma)d\gamma.$$

Equals capacity with effective power loss K=-1.5/ln(5BER).

Review continued

Constellation Restriction

- Restrict $M_D(\gamma)$ to $\{M_0=0,\ldots,M_N\}$.
- Let $M(\gamma)=\gamma/\gamma_K^*$, where γ_K^* is optimized for max rate
- Set $M_D(\gamma)$ to $\max_j M_j$: $M_j \le M(\gamma)$ (conservative)
- Region boundaries are $\gamma_j=M_j\gamma_K^*$, $j=0,\ldots,N$
- Power control maintains target BER



Review continued

Power Adaptation and Average Rate

- Power adaptation:
  - Fixed BER within each region
    - $E_s/N_0=(M_j-1)/K$
    - Channel inversion within a region
  - Requires power increase when increasing $M(\gamma)$

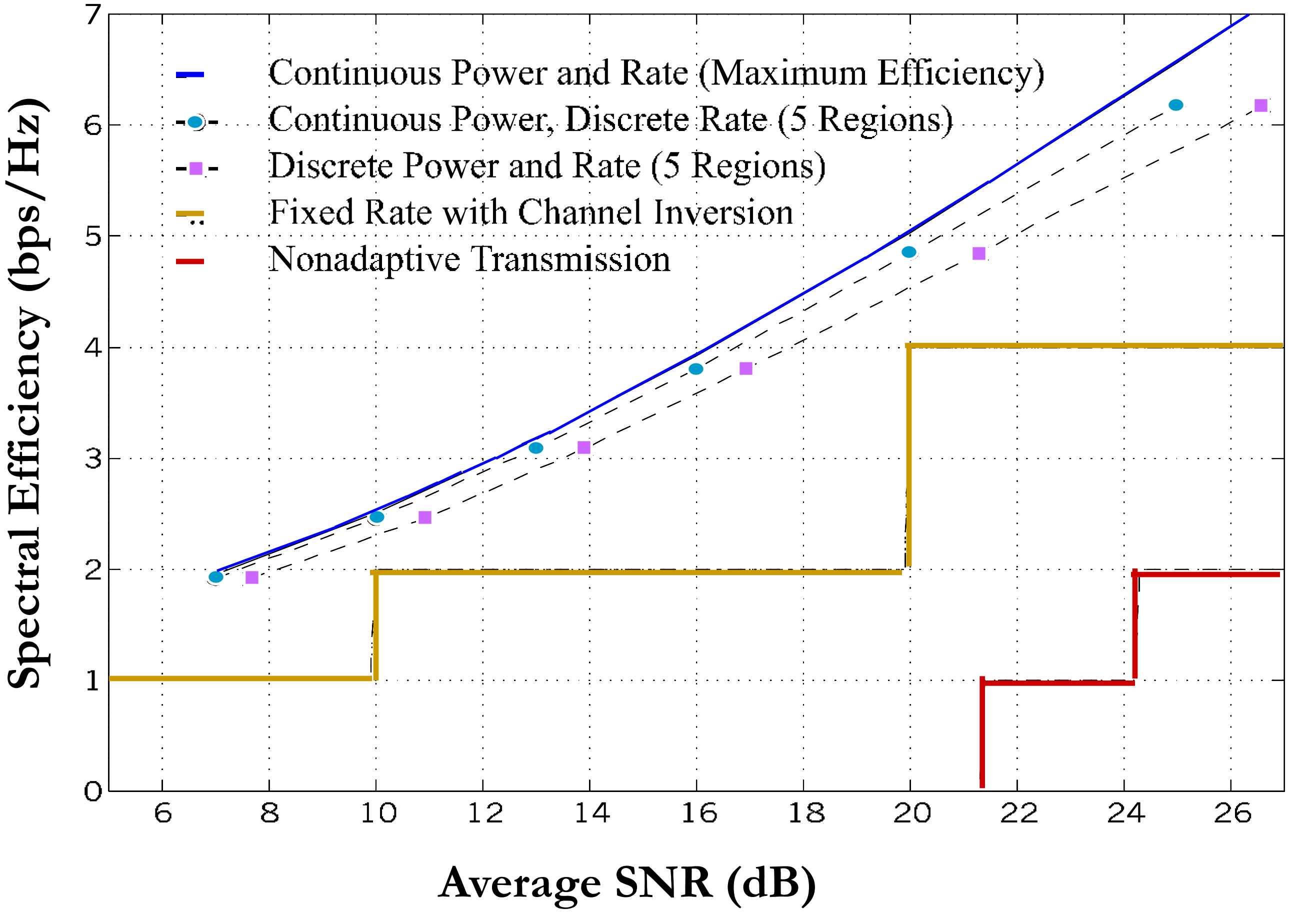
$$\frac{P_j(\gamma)}{P}=\begin{cases}(M_j-1)/(\gamma K) & \gamma_j \le \gamma < \gamma_{j+1},\ j>0 \\ 0 & \gamma<\gamma_1\end{cases}$$

- Average Rate

$$\frac{R}{B}=\sum_{j=1}^{N}\log_2 M_j\, p(\gamma_j \le \gamma < \gamma_{j+1})$$

- Practical Considerations:
  - Update rate/estimation error and delay

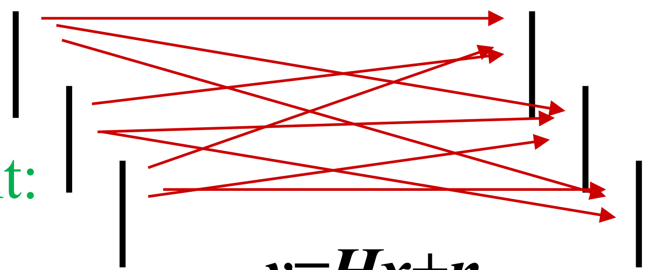
Efficiency in Rayleigh Fading



Multiple Input Multiple Output (MIMO)Systems

- MIMO systems have multiple transmit and receiver antennas (M_t at TX, M_r at RX)

$x=[x_1,\ldots,x_{Mt}]$



$y=[y_1,\ldots,y_{Mr}]$

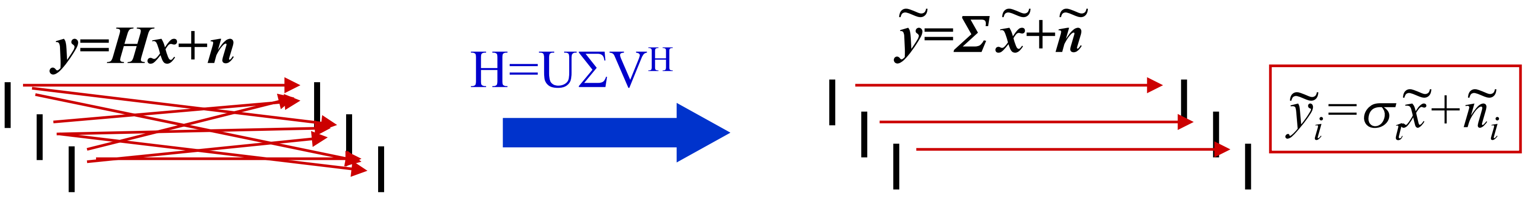
TX power constraint:
$E[\mathbf{x}\mathbf{x}^H]=\rho=P/\sigma^2$

$y=Hx+n$

- Input described by vector x of dimension M_t
- Output described by vector y of dimension M_r
- Channel described by $M_r x M_t$ matrix
- Design and capacity analysis depends on what is known about channel *H* at TX and RX
  - If H unknown at TX, requires vector modulation/demod

MIMO Decomposition

- Decompose channel through transmit precoding ($x=V\tilde{x}$) and receiver shaping ($\tilde{y}=U^H y$)



- Leads to $R_H \le \min(M_t,M_r)$ independent channels with gain σ_i (i^{th} singular value of H) and AWGN

- Independent channels lead to simple capacity analysis and modulation/demodulation design

Capacity of MIMO Systems

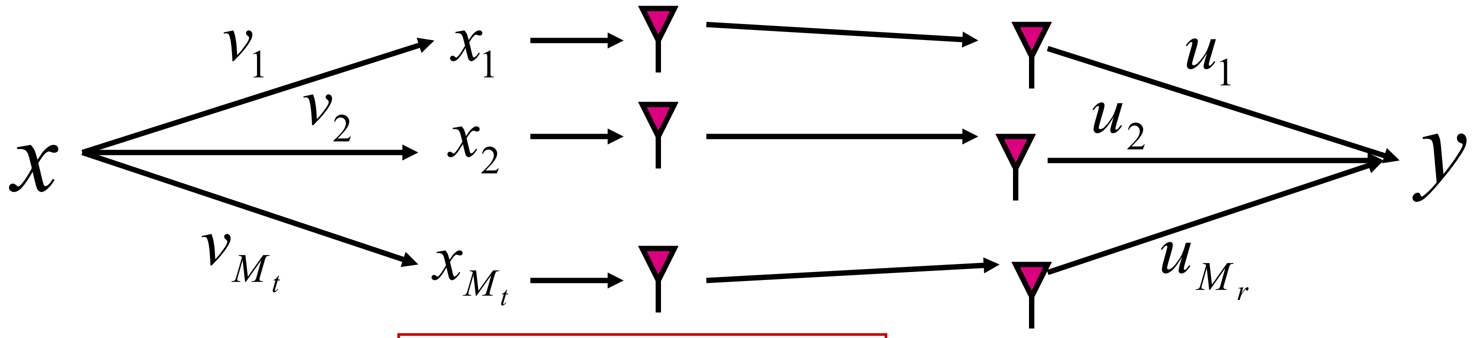
- Depends on what is known at TX and RX and if channel is static or fading
- For static channels
  - With perfect CSI at TX and RX, power water-filling over space is optimal
  - Without transmitter channel knowledge, capacity metric is based on an outage probability
    - P_{out} is the probability that the channel capacity given the channel realization is below the transmission rate.
  - For large arrays, random channel gains converge to static values, $C = \min(M_t, M_r)\, B\log(1+\rho)$; ρ is SNR

MIMO Fading Channel Capacity

- If channel H known, waterfill over space (fixed power at each time instant) or space-time
- Without transmitter channel knowledge, capacity is based on an outage probability
  - P_{out} is the probability that the channel capacity given the channel realization is below the transmission rate.

Beamforming

- Scalar codes with transmit precoding



$$y = \boldsymbol{u}^H \boldsymbol{H} \boldsymbol{v} x + \boldsymbol{u}^H \boldsymbol{n}$$

- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need "sufficient" knowledge for optimality of beamforming

Main Points

- MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain
- With TX and RX channel knowledge, channel decomposes into independent channels
- Capacity of MIMO systems
  - Static channel with TX/RX CSI: sum of capacity on each spatial dimension
  - Static channel without TX CSI: capacity metric is outage. At high SNR/large arrays, capacity increases linearly with the number of TX/RX antennas
  - Fading channel with TX/RX CSI: water-fill power over space or space-time to achieve capacity
- Beamforming transforms MIMO system into a SISO system with TX and RX diversity.
  - Beamform along direction of maximum singular value