Lecture 11 - EE 359: Wireless Communications - Autumn 2017

Diversity

Lecture Outline

- **Diversity Combining Techniques**
- **Selection Combining (SC) and its Performance**
- **Maximal Ratio Combining (MRC)**
- **Performance of MRC with i.i.d. Rayleigh fading**
- **MGF approach to MRC analysis** *(not covered in lecture)*
- **Transmit Diversity**

1. **Diversity Combining Techniques**
   - Selection Combining: largest fading path chosen.
   - Maximal Ratio Combining: all paths cophased and summed with optimal weighting to maximize SNR at combiner output.
   - Equal Gain Combining: all paths cophased and summed with equal weighting.
   - We use space diversity as a reference for analysis; same analysis applies for any mechanism used to obtain independent fading paths.

2. **Array and Diversity Gain**
   - Array gain is the gain in SNR from noise averaging over the multiple antennas. Gain in both AWGN and fading channels.
   - Diversity gain is the change in slope of the probability of error due to diversity. Only applies to fading channels.

3. **Selection Combining (SC) and its Performance**
   - Combiner SNR γ_Σ is the maximum of the branch SNRs.
   - This gives diminishing returns, in terms of power gain, as the number of antennas increases.
   - CDF of γ_Σ easy to obtain, then pdf found by differentiating.
   - Typically get 10-15 dB of gain for 2-3 antennas.

4. **Maximal Ratio Combining (MRC)**
   - Branch weights optimized to maximize output SNR of combiner.
   - Optimal weights are proportional to branch SNR.
   - Resulting combiner SNR γ_Σ is sum of branch SNRs.
   - Distribution obtained by characteristic function analysis (can be hard).

5. **Performance of MRC with i.i.d. Rayleigh fading**

   - For M branch diversity with i.i.d. Rayleigh fading on each branch, γ_Σ is chi-squared with $2M$ degrees of freedom.
   - Can obtain P_{out} and $\overline{P}_s$ from this distribution.
   - For BPSK, get 15 dB gain at 10^{-3} BER. Larger gains obtained at lower BERs.

6. **MGF Approach to MRC Diversity Analysis.** *(not covered in lecture)*

   - Distribution of γ_Σ hard to obtain when fading is not Rayleigh or not identically distributed.
   - Can use alternate Q function representation to greatly simplify $\overline{P}_s$ calculation.
   - Using alternate representation and switching order of integration yields

   $$\overline{P}_s = \frac{\alpha_M}{\pi}\int_0^{\pi/2}\prod_{i=1}^{M}\mathcal{M}_{\gamma_i}\left[\frac{-.5\beta_M}{\sin^2\phi}\right]d\phi,$$

   where $\mathcal{M}_{\gamma_i}$ is the MGF for the distribution of the ith branch SNR γ_i and α_M and β_M are functions of the modulation such that in AWGN, $P_s \approx \alpha_M Q(\sqrt{\beta_M \gamma_s})$.

7. **Transmit Diversity**

   - When channel known at transmitter, similar to receiver diversity. Get same array and diversity gain.
   - When channel unknown at transmitter, for 2 TX antennas can use the Alamouti scheme over two symbol times to obtain full diversity gain, but no array gain. This scheme is part of various wireless standards but is hard to generalize to more than 2 antennas. Alamouti not covered on homeworks or exams.

Main Points

- Both selection combining and MRC significantly reduce the impact of fading.
- SC vs. MRC offer different levels of complexity vs. performance.
- Performance analysis of MRC greatly simplified using MGF approach.
- Transmit diversity for known channel gains has same performance as receiver diversity.