EE359 – Lecture 9 Outline

- Announcements:
  - OHs today12:50-2pm and tomorrow 2:50-3:10 (in Thorton) and before class by appointment.
  - Updated reader (Chapters 1-7) available today or tomorrow.
  - Project proposals due midnight Friday; get early feedback
  - Midterm Feb. 21, 2-4pm (food after), more details next week
    - Email me/TA if you have a conflict
    - Open book/notes. Covers through diversity. No HW that week.
    - Over next week: propose dates for MT review, post practice MTs.
- Linear Modulation Performance in AWGN
- Q-Function representations
- Probability of error in fading
- Outage probability
- Average P_s (P_b)

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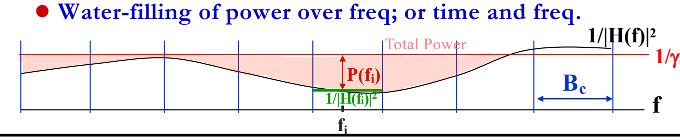
Review of Last Lecture

- Capacity in Flat-Fading: γ known at TX/RX
  - Optimal Adaptation: Power: $1/\gamma_0 - 1/\gamma$, Rate: $B\log_2(\gamma/\gamma_0)$; Depend on $p(\gamma)$ only through γ_0
  - Channel Inversion and Truncated Inversion
    - Received SNR constant; Capacity is $B\log_2(1+\sigma)$ above an outage level associated with truncation
- Capacity of ISI channels
  - Divide wideband channel into narrowband flat-fading subchannels of bandwidth B approximately equal to B_c
    - Each subchannel has NB fading approx. independent from others
  - Water-filling of power over freq; or time and freq.



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Linear Digital Modulation

- Signal over ith symbol period:

$$s(t) = s_{i1} g(t)\cos(2\pi f_c t + \phi_0) - s_{i2} g(t)\sin(2\pi f_c t + \phi_0)$$

  - Pulse g(t) typically Nyquist, assumed square
  - Signal constellation defined by (s_{i1}, s_{i2}) pairs
    - M values for $(s_{i1}, s_{i2}) \Rightarrow \log_2 M$ bits per symbol
    - We focus on MPSK and MQAM
    - MPSK can be differentially encoded
- For MQAM and MPSK, P_s depends on
  - Minimum distance d_{min} *(depends on γ_s)*
  - # of nearest neighbors α_M
  - Approximate expression: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
    - Standard/alternate Q function

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Linear Modulation in Fading

- In fading γ_s and therefore P_s random
- Performance metrics:
  - Outage probability: $p(P_s > P_{target}) = p(\gamma < \gamma_{target})$
  - Average P_s, $\overline{P_s}$:

$$\overline{P_s} = \int_0^\infty P_s(\gamma) p(\gamma) d\gamma$$

  - Combined outage and average P_s

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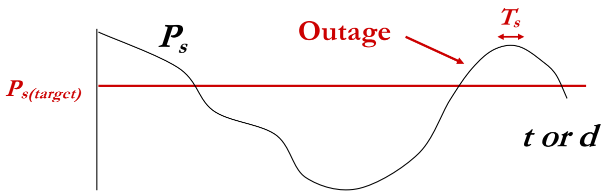
Outage Probability



- Probability that P_s is above target
- Equivalently, probability γ_s below target
- Used when $T_c >> T_s$

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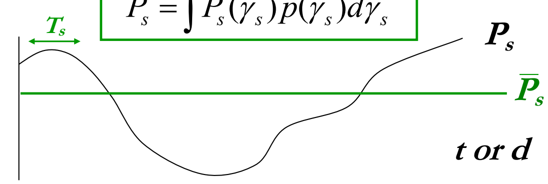
Average P_s

$$\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$$



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Rarely obtain average error probability in closed form
  - Probability in AWGN is Q-function, double infinite integral

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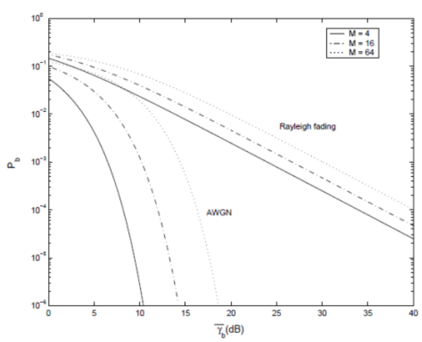
Average Probability of Error (MQAM)



Fading severely degrades performance

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Alternate Q Function Analysis

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

  - Infinite integrand, argument in integral limits
  - Average P_e entails infinite integral over Q(z)
- Craig's representation: $Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)} d\varphi$
  - Very useful in fading and diversity analysis
    - AWGN formula: $P_s(\gamma_s) \cong \alpha Q(\sqrt{\beta\gamma_s})$
    - Fading formula:

$$\overline{P}_s \cong \frac{\alpha}{\pi}\int_0^{\pi/2} \mathcal{M}_{\gamma_s}\left(\frac{-.5\beta}{\sin^2\phi}\right) d\phi$$

M_{γ_s} is MGF of fading pdf of γ_s; α,β, depend on modulation*

*current reader has -.5β=g notation

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Main Points

- P_s approximation in AWGN: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
  - For MPSK, MQAM
- In fading P_s is a random variable, characterized by average value, outage, or combined outage/average
- Fading greatly increases average P_s or required power for a given target P_s with some outage
  - Outage probability based on target SNR in AWGN.
- Need to combat flat fading or waste lots of power

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