EE359 – Lecture 7 Outline

- Announcements:
  - Schedule changes next week
    - No lecture next Tues 2/3.
    - Makeup class: Wed 2/5 11:30-12:50pm w/lunch in Gates B03
  - Project proposals due 2/7; I can provide early feedback
  - MT week of 2/17, 6-8pm (pizza after), poll this week; details soon
- Doppler in Wideband Channels
- Shannon Capacity
- Capacity of Flat-Fading Channels
  - Fading Statistics Known
  - Fading Known at RX
  - Fading Known at TX and RX: water-filling

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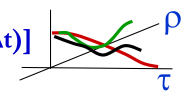
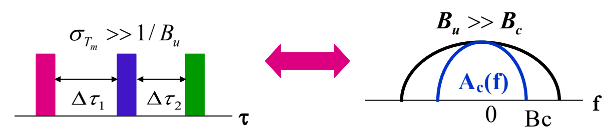
Review of Last Lecture

- Wideband channels: $B_u >> 1/\sigma_{T_m}$
- Scattering Function: $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$
  - Used to characterize $c(\tau,t)$ statistically



- Multipath Intensity Profile:
- Determines average (μ_{T_m}) and rms (σ_{T_m}) delay spread
  - Coherence bandwidth $B_c=1/\sigma_{T_m}$



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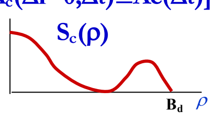
Doppler Power Spectrum

Scattering Function: $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$

- Doppler Power Spectrum: $S_c(\rho)= \mathcal{F}_{\Delta t}[A_c(\Delta f=0,\Delta t)\triangleq Ac(\Delta t)]$

  $A_c(\Delta f,\Delta t)=\mathcal{F}_\tau[A_c(\tau,\Delta t)]$



  - Power of multipath at given Doppler
  - Doppler spread B_d: Max. doppler for which $S_c(\rho)>0$.
  - Coherence time $T_c=1/B_d$: Max time over which $A_c(\Delta t)>0$
    - $A_c(\Delta t)=0$ implies signals separated in time by Δt uncorrelated at RX
- Why do we look at Doppler w.r.t. $A_c(\Delta f=0,\Delta t)$?
  - Captures Doppler associated with a narrowband signal
  - Autocorrelation over a narrow range of frequencies
  - Fully captures time-variations, multipath angles of arrival

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Shannon Capacity

- Defined as channel's maximum mutual information
- Shannon proved that capacity is the maximum error-free data rate a channel can support.
- Theoretical limit (not achievable)
- Channel characteristic
  - Not dependent on design techniques
- In AWGN, $C=B\log_2(1+\gamma)$ bps
  - B is the signal bandwidth
  - $\gamma=P_r/(N_0B)$ is the received signal to noise power ratio

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Capacity of Flat-Fading Channels

- Capacity defines theoretical rate limit
  - Maximum error free rate a channel can support
- Depends on what is known about channel
- Fading Statistics Known
  - Hard to find capacity
- Fading Known at Receiver Only

$$C=\int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma \le B\log_2(1+\bar{\gamma})$$

- Fading known at TX and RX
  - Multiplex optimal strategy over each channel state

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Capacity with Fading Known at Transmitter and Receiver

- For fixed transmit power, same as with only receiver knowledge of fading
- Transmit power $P(\gamma)$ can also be adapted
- Leads to optimization problem

$$C=\max_{P(\gamma):E[P(\gamma)]\le \bar{P}} \int_0^\infty B\log_2\left(1+\frac{\gamma P(\gamma)}{\bar{P}}\right)p(\gamma)d\gamma$$

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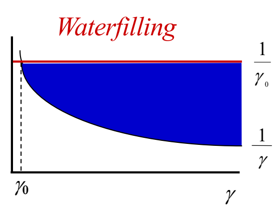
Optimal Adaptive Scheme

- Power Adaptation

$$\frac{P(\gamma)}{\bar{P}}=\begin{cases}\frac{1}{\gamma_0}-\frac{1}{\gamma} & \gamma\ge\gamma_0\\ 0 & \textbf{else}\end{cases}$$



- Capacity

$$\frac{R}{B}=\int_{\gamma_0}^{\infty}\log_2\left(\frac{\gamma}{\gamma_0}\right)p(\gamma)d\gamma.$$

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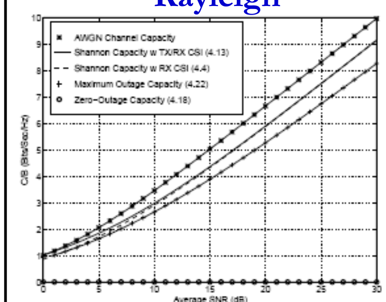
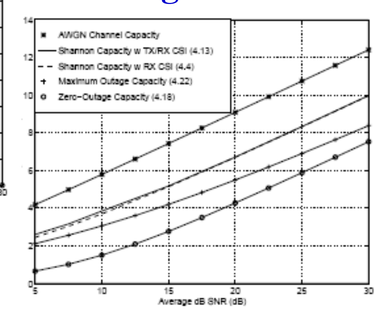
Capacity in Flat-Fading

Rayleigh



Log-Normal



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Main Points

- Doppler spread defines maximum nonzero doppler, its inverse is coherence time
  - Channel decorrelates over channel coherence time
- Fundamental channel capacity defines maximum data rate that can be supported on a channel
- Capacity in fading depends what is known at TX/RX
- Capacity with RX CSI is average of AWGN capacity
- Capacity with TX/RX knowledge requires optimal adaptation based on current channel state
- Almost same capacity as with RX knowledge only

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