Rician and Nakagami Fading. Wideband Fading. Doppler and Delay Spread.

Lecture Outline

- **Rician Fading**
- **Nakagami Fading**
- **Wideband Channel Models**
- **Scattering Function**
- **Multipath Intensity Profile, Delay Spread, and Coherence Bandwidth**
- **Doppler Power Spectrum, Doppler Spread, and Coherence Time**

1. **Rician Fading:**
   - A LOS component leads to a received signal with non-zero mean. The Rician distribution models signal envelope in this case, with K factor dictating the relative power of the LOS component: $p_Z(z) = \frac{z}{\sigma^2}\exp\left[\frac{-(z^2+s^2)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0.$
   - The average received power in the Rician fading is $P_r = \int_0^\infty z^2 p_Z(z)dz = s^2 + 2\sigma^2$.
   - The Rician distribution is often described in terms of a fading parameter K, defined by $K = \frac{s^2}{2\sigma^2}$. The distribution in terms of K is: $p_Z(z) = \frac{2z(K+1)}{P_r}\exp\left[-K - \frac{(K+1)z^2}{P_r}\right] I_0\left(2\sqrt{\frac{K(K+1)z}{P_r}}\right)$.
2. **Nakagami Fading Distribution**
   - Experimental results support a Nakagami distribution for the signal envelope for some environments. Nakagami is similar to Rician, but can model "worse than Rayleigh."
   - Model generally leads to closed-form expressions in BER and diversity analysis.
   - Distribution is $p_Z(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)P_r^m}\exp\left[\frac{-mz^2}{P_r}\right], \quad m \geq .5$. By change of variables, power distribution is $p_{Z^2}(x) = \left(\frac{m}{P}\right)^m \frac{x^{m-1}}{\Gamma(m)}$.
3. **Wideband Channel Models**
   - In wideband multipath channels the individual multipath components can be resolved by the receiver. True if $T_m > 1/B$.
   - If the components can be resolved then they can be combined for diversity gain (e.g. using an equalizer).
4. **Channel Scattering Function:**
   - Typically time-varying channel impulse response $c(\tau, t)$ is unknown, so its wideband model must be characterized statistically.

- Since under our random model with a large number of scatterers, $c(\tau, t)$ is Gaussian. We assume it is WSS, so we only need to characterize its mean and correlation, which is independent of time. Similar to narrowband model, for ϕ_n uniformly distributed, $c(\tau, t)$ has mean zero.
- Autocorrelation of $c(\tau, t)$ is $A_c(\tau_1, \tau_2; \Delta t) = A_c(\tau_1, \tau_2; \Delta t)\delta(\tau_1 - \tau_2) = A_c(\tau; \Delta t)$ since we assume channel response associated with different scatterers is uncorrelated.
- Statistical scattering function defined as $S(\tau, \rho) = \mathcal{F}_{\Delta t}[A_c(\tau, \Delta t)]$. This function measures the average channel gain as a function of both delay τ and Doppler ρ.
- $S(\tau, \rho)$ easy to measure empirically and is used to get average delay spread T_M, rms delay spread σ_τ, and Doppler spread B_d for empirical channel measurements.

5. **Multipath Intensity Profile and Delay Spread**

   - Multipath intensity profile (delay power spectrum) defined as $A_c(\tau; \Delta t = 0) \triangleq A_c(\tau)$, i.e. the autocorrelation relative to delay τ at a fixed time.
   - The average delay μ_{T_m} and rms delay spread σ_{T_m} are defined relative to $A_c(\tau)$. These parameters approximate the maximum delay of nontrivial multipath components.

6. **Coherence Bandwidth**

   - The coherence bandwidth is defined relative to the Fourier transform of $A_c(\tau)$, given by $A_C(\Delta f) = \mathcal{F}[A_c(\tau)]$. Note that $A_C(\Delta f) = A_C(\Delta f, \Delta t = 0)$.
   - Since $A_C(\Delta f)$ is the autocorrelation of a Gaussian process, multipath components separated by Δf_0 are independent if $A_C(\Delta f_0) \approx 0$.
   - By the Fourier transform relationship, the bandwidth over which $A_C(\Delta f)$ is nonzero is roughly $B_c \approx 1/\sigma T_m$ or $B_c \approx 1/\sigma_{T_m}$ (can also add constants to these denominators).
   - B_c defines the coherance bandwidth of the channel, i.e. the bandwidth over which fading is correlated.
   - A signal experiences *frequency selective fading* or *ISI* if its bandwidth exceeds the coherence bandwidth of the channel.

7. **Doppler Power Spectrum, Doppler Spread, and Coherence Time:**

   - Doppler power spectrum is defined with respect to $A_c(\Delta f; \Delta t) = \mathcal{F}_\tau[A_c(\tau, \Delta t)]$.
   - Specifically, the Doppler power spectrum is $S_c(\rho) = \mathcal{F}_{\Delta t}[A_c(\Delta f = 0, \Delta t) \triangleq A_c(\Delta t)]$, which measures channel intensity as a function of Doppler frequency.
   - The maximum value of ρ for which $|S_c(\rho)| > 0$ is called the channel Doppler spread, which is denoted by B_d.
   - By the Fourier transform relationship, $A_c(\Delta t) \approx 0$ for $\Delta t > 1/B_d$. Thus, the channel becomes uncorrelated over a time of $1/B_d$ seconds.
   - We define the channel coherance time as $T_c = 1/B_d$. A deep fade lasts approximately T_c seconds. Hence, if the coherence time greatly exceeds a bit time, the signal experiences *error bursts* lasting T_c seconds.

Main Points

- The signal envelope under narrowband fading with uniform AOA is Rayleigh. Other common distribution are Ricean (when a LOS component exists) and Nakagami.
- Wideband models characterized by scattering function, which measures average channel gain relative to delay and Doppler.
- Scattering function used to obtain key channel characteristics of rms delay spread and Doppler spread, which are important for system design.
- Multipath delay spread defines the maximum delay of significant multipath components. Its inverse is the channel coherence bandwidth. Signals separated in frequency by the coherence bandwidth have independent fading.
- Doppler spread defines the channel's maximum nonzero Doppler. Its inverse is the channel coherence time. Signals separated in time by the coherence time have independent fading.