EE359 – Lecture 6 Outline

- Announcements:
  - HW due tomorrow 4pm
- Review of Last Lecture
- Wideband Multipath Channels
- Scattering Function
- Multipath Intensity Profile
- Doppler Power Spectrum

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Review of Last Lecture

- For $\phi_n \sim U[0,2\pi]$, $r_I(t), r_Q(t)$ zero mean, WSS, with

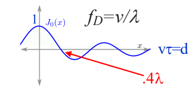
$$A_{r_I}(\tau) = P_r E_{\theta_n}[\cos 2\pi f_{D_n}\tau] = A_{r_Q}(\tau),\ \ f_{D_n} = v\cos\theta_n/\lambda$$

$$A_{r_I,r_Q}(\tau) = P_r E_{\theta_n}[\sin 2\pi f_{D_n}\tau] = -A_{r_I,r_Q}(\tau)$$

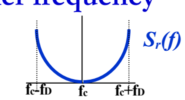
- Uniform AoAs in Narrowband Model
  - In-phase/quad comps have zero cross correlation and

$$A_{r_I}(\tau) = A_{r_Q}(\tau) = P_r J_0(2\pi f_D \tau)$$

Decorrelates over roughly half a wavelength



- PSD maximum at the maximum Doppler frequency
  - PSD used to generate simulation values



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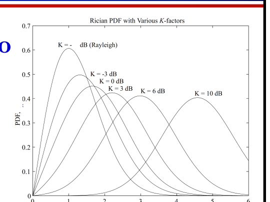
Review Continued:
Signal Envelope Distribution

- CLT approx. leads to Rayleigh distributio (power is exponential*)

To cover today

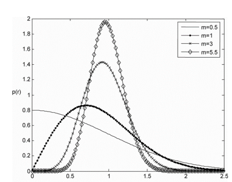
- When LOS component present, Rician distribution is used

$$p_Z(z) = \frac{2z(K+1)}{P_r}\exp\left[-K - \frac{(K+1)z^2}{P_r}\right] I_0\left(2\sqrt{\frac{K(K+1}{P_r}}z\right)$$



Reader correction

- Measurements support Nakagami distribution in some environments
  - Similar to Ricean, but models "worse than Rayleigh"
  - Lends itself better to closed form BER expressions



*Correct in lecture 5 handout; Reader corrections on next slide

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Reader correction: Rayleigh Distribution
(Section 3.2.2, pp. 87-88, correct in 1st Ed.)

- X and Y zero-mean Gaussian, variance σ^2, define Z:

$$Z = \sqrt{X^2 + Y^2}$$

- Signal envelope z(t)=|r(t)|; r(t) has power $P_r = 2\sigma^2$

$$z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)}$$

- Envelope: Z, z(t), and |r(t)| are Rayleigh distributed

$$p_Z(z) = \frac{2z}{P_r}\exp\left[-\frac{z^2}{P_r}\right] = \frac{z}{\sigma^2}\exp\left[-\frac{z^2}{2\sigma^2}\right]$$

- Power: Z^2, $z^2(t)$, and $|r(t)|^2$ are exponentially distributed

$$p_{Z^2}(x) = \frac{1}{P_r}e^{-x/P_r} = \frac{1}{2\sigma^2}e^{-x/2\sigma^2}$$

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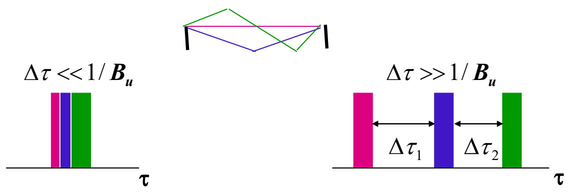
Wideband Channels

- Individual multipath components resolvable
- True when time difference between components exceeds signal bandwidth
  - High-speed wireless systems are wideband for most environments



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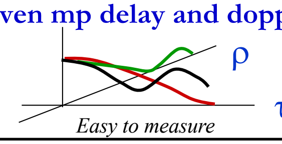
Scattering Function

- Typically characterize c(τ,t) by its statistics, since it is a random process
- Underlying process WSS and Gaussian, so only characterize mean (0) and correlation
- Autocorrelation is $A_c(\tau_1,\tau_2,\Delta t)=A_c(\tau,\Delta t)$
  - Correlation for single mp delay/time difference
- Statistical scattering function:
  - Average power for given mp delay and doppler

$s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$



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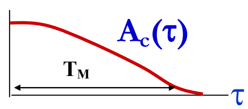
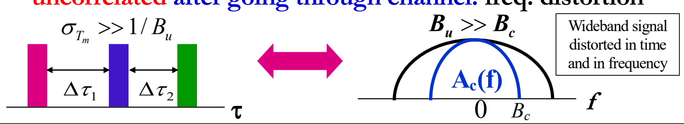
Multipath Intensity Profile



- Defined as $A_c(\tau,\Delta t=0)= A_c(\tau)$
  - Determines average (μ_{T_m}) and rms (σ_{T_m}) delay spread
  - Approximates maximum delay of significant multipath
- Coherence bandwidth $B_c=1/\sigma_{T_m}$
  - Maximum frequency over which $A_c(\Delta f)=F[A_c(\tau)]>0$
  - $A_c(\Delta f)=0$ implies signals separated in freq. by Δf will be uncorrelated after going through channel: freq. distortion



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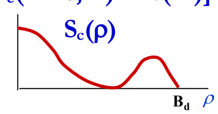
Doppler Power Spectrum

Scattering Function: $s(\tau,\rho)=\mathcal{F}_{\Delta t}[A_c(\tau,\Delta t)]$

- Doppler Power Spectrum: $S_c(\rho)=\mathcal{F}_{\Delta t}[A_c(\Delta f=0,\Delta t)\triangleq Ac(\Delta t)]$

$A_c(\Delta f,\Delta t)=\mathcal{F}_{\tau}[A_c(\tau,\Delta t)]$



  - Power of multipath at given Doppler
  - Doppler spread B_d: Max. doppler for which $S_c(\rho)=>0$.
  - Coherence time $T_c=1/B_d$: Max time over which $A_c(\Delta t)>0$
    - $A_c(\Delta t)=0 \Rightarrow$ signals separated in time by Δt uncorrelated after passing through channel
- Why do we look at Doppler w.r.t. $A_c(\Delta f=0,\Delta t)$?
  - Captures Doppler associated with a narrowband signal
  - Autocorrelation over a narrow range of frequencies
  - Fully captures time-variations, multipath angles of arrival

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Main Points

- Wideband channels have resolvable multipath
  - Statistically characterize c(τ,t) for WSSUS model
  - Scattering function characterizes rms delay and Doppler spread. Key parameters for system design.
- Delay spread defines maximum delay of significant multipath components. Inverse is coherence BW
  - Signal distortion in time/freq. when delay spread exceeds inverse signal BW (signal BW exceeds coherence BW)
- Doppler spread defines maximum nonzero doppler, its inverse is coherence time
  - Channel decorrelates over channel coherence time

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