Shadowing, Combined Path Loss/Shadowing, Data Model Parameters, Statistical Multipath Model

Lecture Outline

- **Log Normal Shadowing**
- **Combined Path Loss and Shadowing**
- **Outage Probability**
- **Model Parameters from Empirical Data**
- **Statistical Multipath Model**

1. **Log-normal Shadowing:**

   - Statistical model for variations in the received signal amplitude due to blockage.
   - The received signal power with the combined effect of path loss (power falloff model) and shadowing is, in dB, given by

   $$P_r(\text{dB}) = P_t(\text{dB}) + 10\log_{10} K - 10\gamma \log_{10}(d/d_r) - \psi(\text{dB}).$$

   - Empirical measurements support the log-normal distribution for ψ:

   $$p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi}\exp\left[-\frac{(10\log_{10}\psi - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right],\ \psi > 0,$$

   where $\xi = 10/\ln 10$, $\mu_{\psi_{dB}}$ is the mean of $\psi_{\text{dB}} = 10\log_{10}\psi$ in dB and $\sigma_{\psi_{dB}}$ is the standard deviation of ψ_{dB}, also in dB.
   - With a change of variables, setting $\psi_{\text{dB}} = 10\log_{10}\psi$, we get

   $$p(\psi_{\text{dB}}) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}}\exp\left[-\frac{(\psi_{\text{dB}} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}}\right], -\infty < \psi_{\text{dB}} < \infty.$$

   - This empirical distribution can be justified by a CLT argument.
   - The autocorrelation based on measurements follows an autoregressive model:

   $$A_\psi(\delta) = \sigma^2_{\psi_{dB}} e^{-\delta/X_c} = \sigma^2_{\psi_{dB}} e^{-v\tau/X_c},$$

   where X_c is the decorrelation distance, which depends on the environment.

2. **Combined Path Loss and Shadowing**

   - Linear Model:

   $$\frac{P_r}{P_t} = K\left(\frac{d}{d_r}\right)^\gamma \psi.$$

   - dB Model:

   $$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma\log_{10}(d/d_r) - \psi_{dB}.$$

   - Average shadowing attenuation: when $K_{\text{dB}} = 10\log_{10} K$ captures average dB shadowing, $\mu_{\psi_{dB}} = 0$, otherwise $\mu_{\psi_{dB}} > 0$ since shadowing causes positive attenuation.

3. **Outage Probability under Path Loss and Shadowing**

   - With path loss and shadowing, the received power at any given distance between transmitter and receiver is random.
   - Leads to a non-circular coverage area around the transmitter, i.e. non-circular contours of constant power above which performance (e.g. in WiFi or cellular) is acceptable.
   - Outage probability $P_{out}(P_{min}, d)$ is defined as the probability that the received power at a given distance d, $P_r(d)$, is below a target P_{min}: $P_{out}(P_{min}, d) = p(P_r(d) < P_{min})$.
   - For the simplified path loss model and log normal shadowing this becomes

$$p(P_r(d) \leq P_{min}) = 1 - Q\left(\frac{P_{min} - (P_t + K_{\text{dB}} - 10\gamma \log_{10}(d/d_r))}{\sigma_{\psi_{dB}}}\right).$$

4. **Model Parameters from Empirical Data:**

   - Constant K_{dB} typically obtained from measurement at distance d_0.
   - Power falloff exponent γ obtained by minimizing the MSE of the predicted model versus the data (assume N samples):

$$F(\gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2,$$

     where $M_{\text{measured}}(d_i)$ is the ith path loss measurement at distance d_i and $M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma \log_{10}(d_i)$. The minimizing γ is obtained by differentiating with respect to γ, setting this derivative to zero, and solving for γ.
   - The resulting path loss model will include average attenuation, so $\mu_{\psi_{dB}} = 0$.
   - Can also solve simultaneously for (K_{dB}, γ) via a least squares fit of both parameters to the data. Using the line equation for each data point y_i that $y_i = mx_i + K_{\text{dB}}$ for $m = -10\gamma$ and $x_i = \log_{10}(d_i)$, the error of the straight line fit is

$$F(K, \gamma) = \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - (mx_i + K_{\text{dB}})]^2,$$

   - The shadowing variance $\sigma^2_{\psi_{dB}}$ is obtained by determining the MSE of the data versus the empirical path loss model with the minimizing $\gamma = \gamma_0$:

$$\sigma^2_{\psi_{dB}} = \frac{1}{N} \sum_{i=1}^{N} [M_{\text{measured}}(d_i) - M_{\text{model}}(d_i)]^2; M_{\text{model}}(d_i) = K_{\text{dB}} - 10\gamma_0 \log_{10}(d_i).$$

5. **Statistical Multipath Model:**

   - At each time instant there are a random number $N(t)$ of multipath signal components.
   - At time t the ith component has a random amplitude $\alpha_i(t)$, angle of arrival $\theta_i(t)$, Doppler shift $f_{D_i} = \frac{v}{\lambda} \cos \theta_i(t)$ and associated phase shift $\phi_{D_i} = \int_t f_{D_i}(t)dt$, and path delay relative to the LOS component $\tau_i(t) = (x_i(t))/c$.
   - Thus, the received signal is given by the following expression, which implies the channel has a time-varying impulse response.

$$r(t) = \Re\left\{\sum_{i=0}^{N(t)} \alpha_i(t) u(t - \tau_i(t)) e^{j(2\pi f_c(t - \tau_i(t)) + \phi_{D_i})}\right\}$$

Main Points

- Shadowing decorrelates over its decorrelation distance, which is on the order of the size of shadowing objects.
- Combined path loss and shadowing leads to outage and non-circular coverage area (cells).
- Path loss and shadowing parameters are obtained from empirical measurements through a least-squares fit.
- Can find path loss exponent γ by a 1-dimensional least-squares-error line fit assuming a fixed value of K_{dB} from one far-field measurement (most common), or find path loss exponent γ and K_{dB} parameters simultaneously through a 2-dimensional least-squares-error line fit.
- Statistical multipath model leads to a time varying channel impulse response