MIMO RX Design, ISI Mitigation Techniques, Multicarrier Modulation and OFDM

Lecture Outline

- **MIMO RX Design**
- **ISI Mitigation Techniques**
- **Multicarrier Modulation**
- **Overlapping Subcarriers and OFDM**

1. **MIMO Receiver Design: Maximum Likelihood Receiver**
   - Optimal MIMO receiver is maximum-likelihood (ML) receiver solves the following problem.
     $$\hat{\mathbf{x}}(\mathbf{y}) = \text{argmin}_{\mathbf{x}\in\mathcal{X}^{\mathbf{M_t}}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^{\mathbf{2}}.$$
   - Must search over all possible $\mathbf{x} \in \mathcal{X}^{\mathbf{M_t}}$, i.e. over all possible transmitted vectors which has exponential complexity in M_t and in the constellation size.
   - Alternate approaches using linear processing of input vector $\mathbf{Ax}$.

2. **Linear MIMO Receivers:**
   - Multiplies $\mathbf{y}$ with a MIMO equalization matrix $\mathbf{A} \in \mathbf{C}^{\mathbf{M_t}\times\mathbf{M_r}}$ to get $\tilde{\mathbf{x}}(\mathbf{y}) \in \mathbf{C}^{\mathbf{M_t}}$.
   - Zero-forcing receiver forces all interference from other symbols to zero:
     $$\tilde{\mathbf{x}}(\mathbf{y}) = \mathbf{H}^{\dagger}\mathbf{y},$$
     where $\mathbf{H}^{\dagger}$ is $\mathbf{H}^{-\mathbf{1}}$ if the matrix is square and invertible, otherwise we use the **pseudo inverse**.
   - When $M_t \leq M_r$, and there are at least M_t linearly independent columns in $\mathbf{H}$ (typical for many propagation environments), the pseudo inverse (Moore-Penrose pseudoinverse) is $\mathbf{H}^{\dagger} = (\mathbf{H}^{\mathbf{H}}\mathbf{H})^{-\mathbf{1}}\mathbf{H}^{\mathbf{H}}$.
   - Complexity of obtaining $\mathbf{H}^{\dagger}$ is roughly cubic in M_t for square matrices. However obtaining $\hat{\mathbf{x}}(\mathbf{y})$ from $\tilde{\mathbf{x}}(\mathbf{y})$ is linear in M_t. ZF can result in significant noise enhancement.
   - L-MMSE receiver: trades off cancellation of interference from other symbols for noise enhancemen with a regularization term that depends on SNR. The solution is
     $$\tilde{\mathbf{x}}(\mathbf{y}) = (\mathbf{H}^{\mathbf{H}}\mathbf{H} + \lambda\mathbf{I})^{-\mathbf{1}}\mathbf{H}^{\mathbf{H}}\mathbf{y} = \mathbf{H}^{\mathbf{H}}(\mathbf{H}\mathbf{H}^{\mathbf{H}} + \lambda\mathbf{I})^{-\mathbf{1}}\mathbf{y},$$
     which reduces to zero forcing in the absence of noise (infinite SNR).
   - Compared to the ML detector, both the ZF and MMSE linear detectors are simpler to implement, but their BER performances are worse.

3. **Sphere Decoder:**

   - A nonlinear technique that allows for better performance than a linear detector and a simple mechanism to trade off between complexity and performance
   - Uses upper triangular decomposition of $\mathbf{H} = \mathbf{QR}$, where $\mathbf{Q}$ is unitary and $\mathbf{R}$ is upper triangular to reduce complexity.
   - Sphere decoder solves the following problem:

$$\hat{\mathbf{x}}(\mathbf{y}) = \mathbf{argmin}_{\mathbf{x}\in\mathcal{X}^{\mathbf{M_t}}, \|\mathbf{Q^H y - Rx}\| \leq \mathbf{r}} \|\mathbf{Q^H y - Rx}\|^{\mathbf{2}}$$

   - Choosing $r = \infty$ gives us the ML decoder. For r finite, the solver can exploit the upper triangular nature of $\mathbf{R}$ to "prune" many candidate solutions (using depth-first-search or breadth-first-search or a combination of the two), thereby reducing the detection complexity significantly.
   - If the sphere decoder finds a valid solution, it is the same solution that the ML detector would have returned.

4. **ISI Countermeasures:**

   - Equalization: signal processing at receiver to remove ISI. Too complex for high-speed systems with large delay spread.
   - Multicarrier modulation: send data over independent subcarriers at slow enough rate such that they experience flat-fading.
   - Spread spectrum modulation: Use properties of spreading codes to remove or coherently combine ISI at receiver.
   - Use directional antennas to reduce delay spread and ISI.

5. **Multicarrier Modulation (MCM):**

   - Mitigates ISI by dividing the transmit bit stream into N substreams.
   - Each substream modulated by a separate subcarrier with signal bandwidth B/N.
   - N is made sufficiently large so that $B/N < B_c$, so substreams experience flat-fading.
   - MCM can be implemented using frequency division multiplexing.

6. **Overlapping Subcarriers**

   - More bandwidth-efficient implementation (OFDM)than MCM overlaps the transmitted substreams such that they can be separated at the receiver.
   - For rectangular pulses, minimum required separation is B/N. Can be less if phases of subcarriers are aligned.

Main Points

- MIMO RX design trades complexity for performance. ML detector is optimal but exponentially complex. Linear decoders enhance noise. Sphere decoders allow performance vs. complexity tradeoff via radius; most common technique in practice.
- ISI typically mitigated by equalization, multicarrier modulation, spread spectrum, or antenna techniques. Equalization not used in current wireless standards due to complexity.
- Multicarrier modulation splits data into narrowband (flat-fading) substreams.
- Multicarrier modulation made more bandwidth efficient by overlapping subchannels.