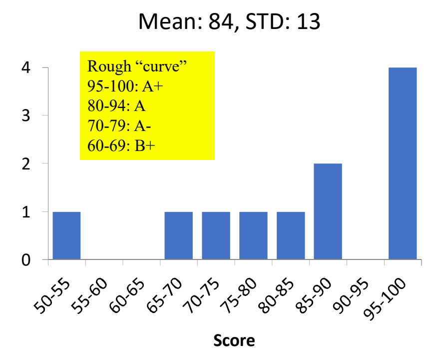
# EE359 – Lecture 15 Outline

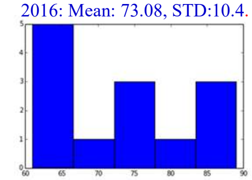
- Announcements:
  - HW posted, due Friday
  - MT exam grading done
    - Can pick up after class or from Dash
  - Makeup lecture next week on Monday (not Wednesday)
- MIMO Fading Channel Capacity
- Massive MIMO
- MIMO Beamforming
- Diversity/Multiplexing Tradeoffs
- MIMO Receiver Design

1

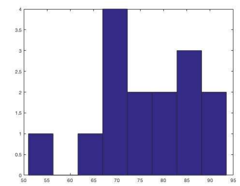
# Midterm Grade Distribution




2016: Mean: 73.08, STD:10.4.

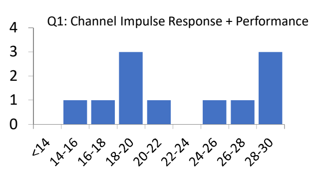


2017: Mean: 76, STD: 11



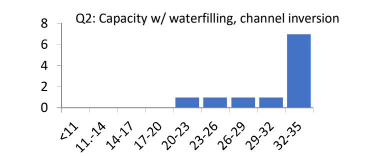
2

# Grade breakdown by problem and common mistakes

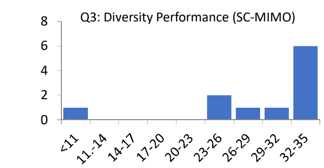



1c. The effect of ISI on this channel was not considered.

1d. The outage probability based on an SNR threshold for average BER due to Rayleigh fading was not properly computed




2.b. and 2.c: Maximum outage capacity under truncated inversion mistaken for channel capacity under channel inversion. Also for 2c, Pout is minimized if Pout = 0.

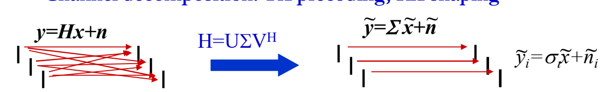



3a: Choice of i;j should maximize $|h_{ij}|^2$, not $h_{ij}$

3

# Review of Last Lecture

- MIMO systems have multiple TX and RX antennas
  - System model defined via matrices and vectors
  - Channel decomposition: TX precoding, RX shaping




- Capacity of MIMO Systems
  - Depends on what is known at TX/RX and if channel is static or fading
  - For static channel with perfect TX/Rx CSI, water-fill over space:

$$C = \max_{P_i:\sum_i P_i \le P} \sum_{i=1}^{R_H} B\log_2\left(1+\frac{\sigma_i^2 P_i}{\sigma^2}\right) = \max_{P_i:\sum_i P_i \le P} \sum_{i=1}^{R_H} B\log_2\left(1+\frac{P_i\gamma_i}{P}\right)$$

  - Without transmitter channel knowledge, capacity metric is based on an outage probability
    - $P_{out}$ is the probability that the channel capacity given the channel realization is below the transmission rate.

4

# MIMO Fading Channel Capacity

- If channel H known, waterfill over space (fixed power at each time instant) or space-time
- Capacity without TX CSI:
  - General expression for AWGN MIMO capacity

$$C = \max_{\mathbf{R_x}:\mathrm{Tr}(\mathbf{R_x})=\rho} B\log_2\det[\mathbf{I}_{M_r} + \mathbf{H}\mathbf{R_x}\mathbf{H}^H] = \sum_{i=1}^{R_\mathbf{H}} B\log_2\left(1+\frac{\gamma_i}{M_t}\right) \qquad \boxed{\gamma_i = \sigma_i^2\rho}$$

  - Without TX CSI, send equal power at each TX antenna ($R_x=(\rho/M_t)I_{M_t}$); capacity based on outage probability
  - $P_{out}$ is probability that channel capacity given the channel realization is below the transmission rate C.

$$P_{\mathrm{out}} = p\left(\mathbf{H} : B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] < R\right)$$

5

# Massive MIMO


$$C = B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right]$$

- For fixed $M_r$, singular values converge to a constant as $M_t$ grows large: $\lim_{M_t\to\infty}\frac{1}{M_t}\mathbf{H}\mathbf{H}^H = \mathbf{I}_{M_r}$

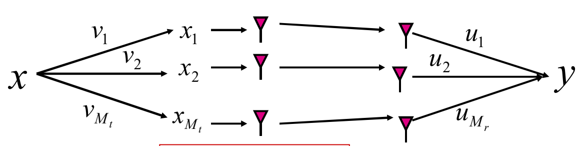
$$\boxed{\lim_{M_t\to\infty} B\log_2\det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] = B\log_2\det\left[\mathbf{I}_{M_r} + \rho\mathbf{I}_{M_r}\right] = M_r B\log_2(1+\rho).}$$

- Capacity grows linearly with $M=\min(M_t,M_r)$
  - Same is true for high SNR and finite $M_t,M_r$

6

# Beamforming

- Scalar codes with transmit precoding



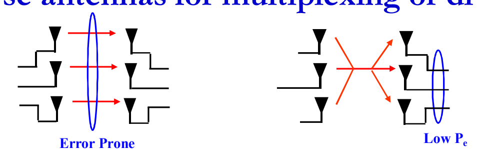
$$\boxed{y = u^H H v x + u^H n}$$

- Transforms system into a SISO system with diversity.
  - Array and diversity gain
  - Greatly simplifies encoding and decoding.
  - Channel indicates the best direction to beamform
  - Need “sufficient” knowledge for optimality of beamforming

7

# Diversity vs. Multiplexing

- Use antennas for multiplexing or diversity

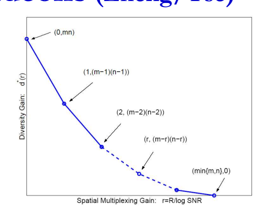



- Diversity/Multiplexing tradeoffs (Zheng/Tse)

$$\lim_{SNR\to\infty}\frac{\log P_e(SNR)}{\log SNR} = -d$$

$$\lim_{SNR\to\infty}\frac{R(SNR)}{\log SNR} = r$$

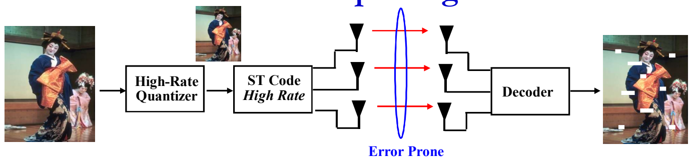
$$\boxed{d^*(r) = (M_t - r)(M_r - r)}$$



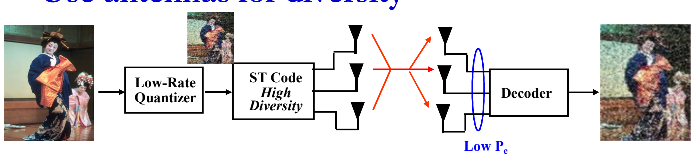

8

# How should antennas be used?

- Use antennas for multiplexing:




- Use antennas for diversity



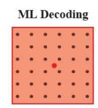

**Depends on end-to-end metric:** ***Solve by optimizing app. metric***

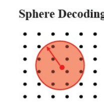
9

# MIMO Receiver Design

- Optimal Receiver:
  - Maximum likelihood: finds input symbol most likely to have resulted in received vector
  - Exponentially complex # of streams and constellation size
- Linear Receivers
  - Zero-Forcing: forces off-diagonal elements to zero, enhances noise
  - Minimum Mean Square Error: Balances zero forcing against noise enhancement
- Sphere Decoder:
  - Only considers possibilities within a sphere of received symbol.
    - If minimum distance symbol is within sphere, optimal, otherwise null is returned

$$\hat{x} = \arg\min_x |y - Hx|^2$$




$$\hat{x} = \arg\min_{x:|Q^H y - Rx| < r} |Q^H y - Rx|^2$$

10

# Main Points

- Capacity of fading MIMO systems
  - With TX and RX channel knowledge, water-fill power over space or space-time to achieve capacity
  - Without TX CSI, outage is the capacity metric
- For massive MIMO or high SNR, capacity scales as $\min(M_t,M_r)$
- Beamforming transforms MIMO system into a SISO system with TX and RX diversity.
  - Beamform along direction of maximum singular value
- MIMO introduces diversity/multiplexing tradeoff
  - Optimal use of antennas depends on application
- MIMO RX design trades complexity for performance
  - ML detector optimal - exponentially complex
  - Linear receivers balance noise enhancement against stream interference
  - Sphere decoding provides near ML performance with linear complexity

11