Discrete Rate Adaptation.
MIMO Communications and Capacity.

Lecture Outline

- **Discrete Rate Adaptation in Adaptive MQAM**
- **MIMO Systems**
- **MIMO Channel Decomposition**
- **MIMO Channel Capacity**
- **Massive MIMO Channel Capacity**

1. **Discrete Rate Adaptation in Adaptive MQAM**

   - Constellation restricted to finite set $\{M_0 = 0, M_1, \ldots, M_{N-1}\}$
   - Divide the fading range of γ into N discrete fading regions R_j.
   - Within each region "conservatively" assign constellation $M_j : M_j \leq M(\gamma) \leq M_{j+1}$, where $M(\gamma) = \gamma/\gamma_K^*$ for some optimized γ_K^*.
   - Power control based on channel inversion: maintains constant BER within region R_j. maintain a fixed BER for the constellation $M_j > 0$ using the power adaptation policy $\frac{P_j(\gamma)}{P} = (M_j - 1)/(\gamma K)$ for $M_j < \gamma/\gamma_K^* \leq M_{j+1}$ and zero otherwise.
   - Rate is then
   $$\frac{R}{B} = \sum_{j=1}^{N-1} \log_2(M_j) p \left(M_j \leq \frac{\gamma}{\gamma_K^*} < M_{j+1} \right).$$
   - Using large enough constellation set results in near-optimal performance.
   - Additional penalty of 1.5-2 dB if each constellation restricted to 1 transmit power.

2. **MIMO Systems**

   - MIMO systems have multiple antennas at the transmitter and receiver.
   - The antennas can be used for capacity gain and/or diversity gain.
   - MIMO system design and analysis complex since it requires vector signal processing.
   - The performance and complexity of MIMO systems depends on what is known about the channel at both the transmitter and receiver

3. **MIMO Channel Decomposition**

   - With perfect channel estimates at the transmitter and receiver, the MIMO channel decomposes into $R_{\mathbf{H}}$ independent parallel channels, where $R_{\mathbf{H}}$ is the rank of the channel matrix ($\min(M_t, M_r)$ for M_t transmit and M_r receive antennas under rich scattering).
   - With this decomposition there is no need for vector signal processing.
   - Decomposition is obtained by transmit precoding and receiver shaping.

4. **MIMO Channel Capacity: Static Channels**

   - Capacity depends on whether the channel is static or fading, and what is known about the channel at the transmitter and receiver.
   - For a static channel known at the transmitter and receiver capacity is given by

   $$C = \max_{P_i : \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{\sigma_i^2 P_i}{\sigma_n^2}\right) = \max_{P_i : \sum_i P_i \leq P} \sum_i B \log_2 \left(1 + \frac{P_i \gamma_i}{P}\right).$$

   This leads to a water-filling power allocation in space.
   - Without transmitter knowledge, outage probability is the right metric for capacity.
   - In the limit of a large antenna array (Massive MIMO), even without TX CSI, random matrix theory dictates that the singular values of the channel matrix converge to the same constant. Hence, the capacity of each spatial dimension is the same, and the total system capacity is $C = \min(M_t, M_r) B \log(1+\rho)$. So capacity grows linearly with the size of the antenna arrays in Massive MIMO systems.

5. **MIMO Channel Capacity: Fading Channels**

   - In fading, if the channel is unknown at transmitter, uniform power allocation is optimal, but this leads to an outage probability since the transmitter doesn't know what rate to transmit at:

   $$P_{out} = p\left(\mathbf{H} : B \log_2 \det \left[\mathbf{I}_{M_r} + \frac{\rho}{M_t} \mathbf{H}\mathbf{H}^H\right] > C\right).$$

   - Capacity with both transmitter and receiver knowledge of the fading is the average of the capacity for the static channel, with power allocated either by an instantaneous or average power constraint. Under the instantaneous constraint power is optimally allocated over the spatial dimension only. Under the average constraint it is allocated over both space and time.
   - Massive MIMO: When the number of TX (or RX) antennas is large, the channel becomes "static" due to the law of large numbers. Specifically

   $$\lim_{M_t \to \infty} \frac{1}{M_t} \mathbf{H}\mathbf{H}^{\mathbf{H}} = \mathbf{I}_{\mathbf{M_r}}.$$

   The MIMO channel capacity then becomes

   $$\lim_{M_t \to \infty} I(\mathbf{x}; \mathbf{y}) = \lim_{\mathbf{M_t} \to \infty} \mathbf{B} \log_{\mathbf{2}} \det \left[\mathbf{I}_{\mathbf{M_r}} + \frac{\rho}{\mathbf{M_t}} \mathbf{H}\mathbf{H}^{\mathbf{H}}\right] = \mathbf{B} \log_{\mathbf{2}} \det \left[\mathbf{I}_{\mathbf{M_r}} + \rho \mathbf{I}_{\mathbf{M_r}}\right] = \mathbf{M_r} \mathbf{B} \log_{\mathbf{2}}(\mathbf{1} + \rho).$$

   Defining $M = \min(M_t, M_r)$, this implies that as M grows large, the MIMO channel capacity in the absence of TX CSI approaches $C = MB \log_2(1+\rho)$ for ρ the SNR, and hence grows linearly in M.

Main Points

- Restricting the size of the constellation set in adaptive modulation leads to negligible performance loss. MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain.

- With both TX and RX CSI, multiple antennas at both transmitter and receiver lead to independent parallel channels.

- With TX and RX CSI, static channel capacity is the sum of capacity on each spatial dimension.
- Without TX CSI, use outage as capacity metric.
- For large arrays, random gains become static, and capacity increases linearly with the number of TX/RX antennas.
- With TX and RX CSI, capacity of MIMO fading channel uses waterfilling in space or space/time - leads to $\min(M_t, M_r)$ capacity gain. Same gain under Massive MIMO.