EE359 – Lecture 14 Outline

- Announcements:
  - MT today, 2-4pm, 103 Hewlitt, pizza afterwards
  - HW posted today, due next Friday
- Discrete-Rate Adaptive Modulation
- Introduction to MIMO Communications
- MIMO Channel Decomposition
- MIMO Channel Capacity
- MIMO Beamforming

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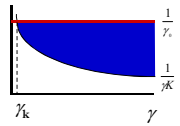
Review of Last Lecture

- Introduction to adaptive modulation
  - Vary different parameters of modulation relative to fading
- Variable-rate variable-power MQAM
  - Maximize average throughput by changing rate and power
  - Optimal power adaptation is water-filling

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma_K} & \gamma \geq \frac{\gamma_0}{K} = \gamma_K \\ 0 & \text{else} \end{cases}$$



- Optimal rate adaptation:

$$\frac{R}{B} = \int_{\gamma_K}^{\infty} \log_2\left(\frac{\gamma}{\gamma_K}\right) p(\gamma) d\gamma.$$

Equals capacity with effective power loss K=-1.5/ln(5BER).

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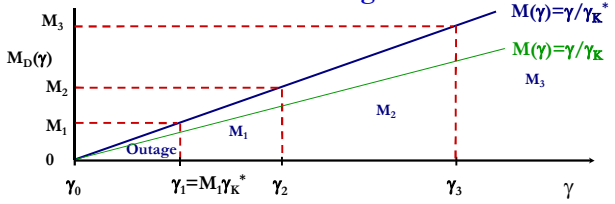
Review continued

Constellation Restriction

- Restrict $M_D(\gamma)$ to $\{M_0=0,\ldots,M_N\}$.
- Let $M(\gamma)=\gamma/\gamma_K^*$, where γ_K^* is optimized for max rate
- Set $M_D(\gamma)$ to $\max_j M_j$: $M_j \leq M(\gamma)$ (conservative)
- Region boundaries are $\gamma_j=M_j\gamma_K^*$, j=0,…,N
- Power control maintains target BER



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Power Adaptation and Average Rate

- Power adaptation: Fixed BER within each region
  - $E_s/N_0=(M_j-1)/K$
  - Channel inversion within a region
  - Requires power increase when increasing M(γ)

$$\frac{P_j(\gamma)}{P} = \begin{cases} (M_j - 1)/(\gamma K) & \gamma_j \leq \gamma < \gamma_{j+1}, j > 0 \\ 0 & \gamma < \gamma_1 \end{cases}$$

- Average Rate

$$\frac{R}{B} = \sum_{j=1}^{N} \log_2 M_j p(\gamma_j \leq \gamma < \gamma_{j+1})$$

- Practical Considerations (not covered in lecture):
  - Cannot update more than every 10-100 symbols
  - Estimation error/delay leads to irreducible error floor

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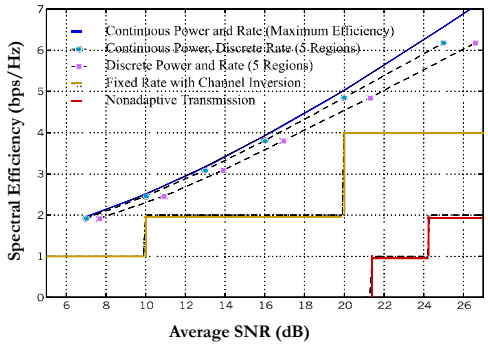
Efficiency in Rayleigh Fading



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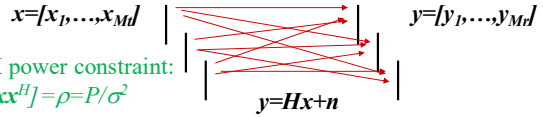
Multiple Input Multiple Output (MIMO)Systems

- MIMO systems have multiple transmit and receiver antennas (M_t at TX, M_r at RX)



TX power constraint: $E[xx^H]=\rho=P/\sigma^2$

- Input described by vector x of dimension M_t
- Output described by vector y of dimension M_r
- Channel described by $M_r x M_t$ matrix
- Design and capacity analysis depends on what is known about channel H at TX and RX
  - If H unknown at TX, requires vector modulation/demod

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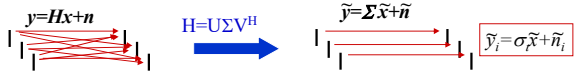
MIMO Decomposition

- Decompose channel through transmit precoding ($x=V\tilde{x}$) and receiver shaping ($\tilde{y}=U^H y$)



- Leads to $R_H \leq \min(M_t, M_r)$ independent channels with gain σ_i (i^{th} singular value of H) and AWGN
- Independent channels lead to simple capacity analysis and modulation/demodulation design

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MIMO Fading Channel Capacity

- If channel H known, waterfill over space (fixed power at each time instant) or space-time
- Without transmitter channel knowledge, capacity is based on an outage probability
  - P_{out} is probability that channel capacity given the channel realization is below the transmission rate C

$$p_{out} = p\left(\mathbf{H} : B\log_2 \det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] < C\right)$$

- Massive MIMO: random channel gains converge to static values: C = $\min(M_t,M_r)$ Blog(1+ρ); ρ is SNR

$$\lim_{M_t\to\infty} B\log_2 \det\left[\mathbf{I}_{M_r} + \frac{\rho}{M_t}\mathbf{H}\mathbf{H}^H\right] = B\log_2 \det\left[\mathbf{I}_{M_r} + \rho\mathbf{I}_{M_r}\right] = M_r B\log_2(1+\rho)$$

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Main Points

- Discretizing the constellation size in adaptive MQMA results in negligible performance loss.
  - Constellations cannot be updated faster than 10s to 100s of symbol times: OK for most dopplers.
  - Estimation error/delay causes error floor
- MIMO systems exploit multiple antennas at both TX and RX for capacity and/or diversity gain
- With TX/RX CSI, decomposes into independent channels
- Capacity of MIMO systems
  - Static channel with TX/RX CSI: sum of capacity on each spatial dimension
  - Static channel without TX CSI: capacity metric is outage.
  - Fading channel with TX/RX CSI: water-fill power over space or space-time to achieve capacity
  - With only RX CSI, capacity metric is outage.
  - Massive MIMO: $C = \min(M_t, M_r)\, B\log(1+\rho)$

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