EE359 – Lecture 10 Outline

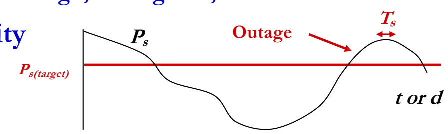
- **Announcements:**
  - Project proposals due tomorrow midnight (post, email link)
  - New reader available tonight
  - Midterm will be Feb. 21, 2-4pm
    - No HW that week, may extend next week's HW deadline
    - Exam open book/notes, covers thru Chp. 7.
    - Midterm review date/time TBD
    - SCPD students can take exam on campus or remotely
    - More MT announcements next week (practice MTs)
- MGF approach for average P_s
- Combined average and outage P_s
- Doppler and delay spread effect on error probability
- Introduction to diversity
- Combining techniques

1

Review of Last Lecture

- Focus on linear modulation
- P_s approximation in AWGN: $P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right)$
  - Nearest neighbor error dominates $Q\left(\sqrt{\frac{d^2_{s_i s_j}}{N_0 B}}\right) >> Q\left(\sqrt{\frac{d^2}{N_0 B}}\right)$ for $d_{s_i s_j} < d$
- Probability of error in fading is random
  - Characterized by outage, average Ps, combination
- Outage probability

Used when $T_c >> T_s$

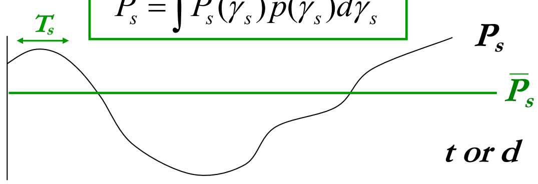


  - Probability P_s is above target; Probability γ_s below target
- Fading severely degrades performance

2

Review Continued: Average P_s

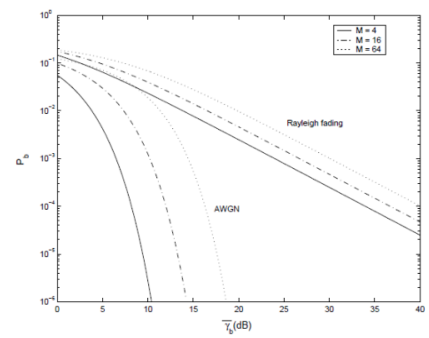
$$\overline{P}_s = \int P_s(\gamma_s)p(\gamma_s)d\gamma_s$$



- Expected value of random variable P_s
- Used when $T_c \sim T_s$
- Error probability much higher than in AWGN alone
- Rarely obtain average error probability in closed form
  - Probability in AWGN is Q-function, double infinite integral

3

Average Probability of Error



Fading severely degrades performance

4

Alternate Q Function Analysis

- Traditional Q function representation

$$Q(z) = p(x > z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx, \quad x \sim N(0,1)$$

  - Infinite integrand, argument in integral limits
  - Average P_e entails infinite integral over Q(z)
- Craig's representation: $Q(z) = \frac{1}{\pi}\int_0^{\pi/2} e^{-z^2/(\sin^2\varphi)} d\varphi$
  - Very useful in fading and diversity analysis
    - AWGN formula: $P_s(\gamma_s) \cong \alpha Q(\sqrt{\beta\gamma_s})$
    - Fading formula:

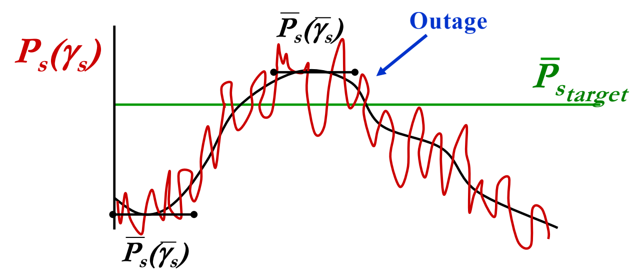
$$\overline{P}_s \cong \frac{\alpha}{\pi}\int_0^{\pi/2} \mathcal{M}_{\gamma_s}\left(\frac{-.5\beta}{\sin^2\phi}\right) d\phi$$

M_{γ_s} is MGF of fading pdf of γ_s; α,β, depend on modulation*

*current reader has .5β=g notation

5

Combined outage and average P_s

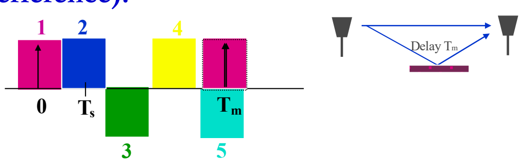


- Used in combined shadowing and flat-fading
- $\overline{P}_s$ varies slowly, locally determined by flat fading
- Declare outage when $\overline{P}_s$ above target value

6

Delay Spread (ISI) Effects

- Delay spread exceeding a symbol time causes ISI (self interference).



- ISI leads to irreducible error floor: $\overline{P}_{b,floor} \approx (\sigma_{T_m}/T_s)^2$
  - Increasing signal power increases ISI power
- ISI imposes data rate constraint: $T_s >> T_m$ ($R_s << B_c$)

$$R \leq \log_2(M) \times \sqrt{\overline{P}_{b,floor}/\sigma^2_{T_m}}$$

7

Doppler Effects

Chap. 6.4 Cover in HW, not lecture

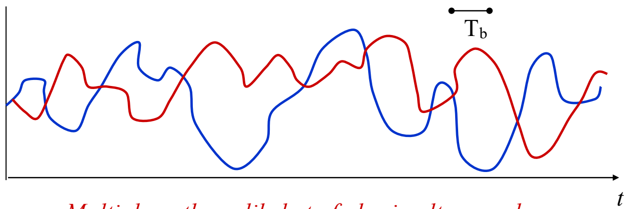
- High doppler causes channel phase to decorrelate between symbols
- Leads to an irreducible error floor for differential modulation
  - Increasing power does not reduce error
- Error floor depends on $f_D T_b$ as

$$P_{floor} = \frac{1 - J_0(2\pi f_D T_b)}{2} \approx .5(\pi f_D T_b)^2$$

8

Introduction to Diversity

- Basic Idea
  - Send same bits over independent fading paths
    - Independent fading paths obtained by time, space, frequency, or polarization diversity
  - Combine paths to mitigate fading effects



Multiple paths unlikely to fade simultaneously

9

Main Points

- Fading greatly increases average P_s or required power for a given target P_s with some outage
- Alternate Q function approach simplifies P_s calculation, especially its average value in fading
  - Average P_s becomes a Laplace transform.
- In fast/slow fading, outage due to shadowing, probability of error averaged over fast fading pdf
- Need to combat flat fading or waste lots of power
  - Adaptive modulation and diversity are main techniques to combat flat fading: adapt to fading or remove it
- Delay spread causes irreducible error floor at high data rates
  - Doppler causes irreducible error floor at low data rates
- Diversity overcomes fading effects by combining fading paths
  - Typically entails penalty in rate, bandwidth, complexity, or size.

10