EE359, Wireless Communications, Winter 2020
Homework 2: Due 4 PM Friday, January 24

Please refer to the homework page on the website (ee359.stanford.edu/homework) for guidelines.

1. (25 pts) **Coverage area formula** (Problems 2-18 & 2-19 adapted) The coverage area is an important metric of the quality of cellular service and can be defined as the expected fraction of a two dimensional area which does not experience outage. In this question we derive an expression for the coverage area based on the free space path loss (with unity gains on transmitter and receiver, frequency 900 MHz, transmit power of 80 mW) and a log normal shadowing model with mean 10 dB and shadowing standard deviation $\sigma_{\psi_{\mathrm{dB}}} = 2$ dB.

   The material in Section 2.7.3 of the textbook, not covered in class, will be used in solving this question.

   (a) (15 pts) Using integration by parts, derive equation (2.48) from (2.46). Evaluate the cell coverage area for the free space path loss parameters given above for a cell radius of 100 m, assuming a $P_{\min}$ of -60 dBm.

   (b) (5 pts) What is the coverage area in (a) if the decorrelation distances (defined in equation (2.38)) are

      i. 1 m,

      ii. 1 km?

   (c) (5 pts) Do your answers in part (b) change if you know that a user close to the transmitter at the center of the cell is in outage? Why or why not?

2. (10 pts) **Time varying channel impulse** (Problem 3-1): Consider a two-ray channel consisting of a direct ray plus a ground-reflected ray, where the transmitter is a fixed base station at height h and the receiver is mounted on a truck (also at height h). The truck starts next to the base station and moves away at velocity v. Assume that signal attenuation on each path follows a free-space path-loss model. Find the time-varying channel impulse at the receiver for transmitter-receiver separation $d = vt$ sufficiently large for the length of the reflected ray to be approximated by $d_{11} + d'_{12} \approx d + \frac{2h^2}{d}$. Please assume the reflection coefficient $R = -1$, and all antenna gains are unity.

3. (15 pts) **On fading distributions** (Problem 3-5):Prove, for X and Y independent zero-mean Gaussian random variables with variance σ^2, that $Z = |X + jY|$ is Rayleigh distributed (i.e. verify distribution of the amplitude) and that the distribution of Z^2 is exponential.

4. (10 pts) **Average signal power** (Problem 3-7) Assume an application that requires a power outage probability of .01 for the threshold $P_0 = -80$ dBm. For Rayleigh fading, what value of the average signal power is required?

5. (20 pts) **Generalized Two-Ray fading model** (Problem 3-10): The Generalized Two-Ray (GTR) fading model assumes that the received signal consists of two dominant components and many other low power diffuse multipath components. The received signal V_r can be modeled as,

$$V_r = V_1 e^{j\phi_1} + V_2 e^{j\phi_2} + X + jY,$$

where V_1 and V_2 are non-negative constants, $\phi_1, \phi_2 \overset{\text{i.i.d.}}{\sim} \mathcal{U}[0, 2\pi)$, impling the phases of the dominant components are uniformly distributed, and $X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$, implying the diffuse components arises from a complex Gaussian distribution. Note that the GTR model reduced to the Rician fading model if $V_1 > 0, V_2 = 0$ and Rayleigh fading if $V_1 = V_2 = 0$. The model is parametrized in terms of K, Δ where

$$K = \frac{V_1^2 + V_2^2}{2\sigma^2}, \qquad \Delta = \frac{2V_1V_2}{V_1^2 + V_2^2}.$$

(a) (5 pts) Find the range of parameters K and Δ.

(b) (5 pts) Interpret parameters K and Δ by describing the fading behavior modeled at the four extreme ends of the ranges found in the previous part ($\{K_{\min}, \Delta_{\min}\}$, $\{K_{\min}, \Delta_{\max}\}$, $\{K_{\max}, \Delta_{\min}\}$, $\{K_{\max}, \Delta_{\max}\}$)

(c) (5 pts) Suppose the phase difference between the two dominant components is constant, that is, $\phi_1 - \phi_2 = \alpha$. In this case, show that the received signal can equivalently be expressed as consisting of one dominant component and the multiple diffuse low power multipath components. This implies that the GTR model reduces to the Rician fading model when the phase difference between the dominant components in constant. Find the parameter $\bar{K}$, the fading parameter (eq. 3.37) of the equivalent Rician model, as a function of α, Δ and K.

(d) (5 pts) Show that the distribution of $\alpha = \phi_1 - \phi_2$ is $\alpha \sim \mathcal{U}[0, 2\pi)$ when $\phi_1, \phi_2 \overset{\text{i.i.d.}}{\sim} \mathcal{U}[0, 2\pi)$. Use this to show that

$$f_{\text{GTR}}(r \mid K, \Delta) = \frac{1}{2\pi} \int_0^{2\pi} f_{\text{Rice}}(r \mid \bar{K}(\alpha, \Delta, K)) \, \mathrm{d}\alpha,$$

where $f_H(r \mid \Sigma)$ is the PDF of the received signal amplitude of channel fading model H with parameters Σ. Note that this expression is a generalization of the finding of part c), where the GTR reduces to the Rician fading model for fixed α.

6. (20 pts) **Macrodiversity** (Problem 3-11): In order to improve the performance of cellular systems, multiple base stations can receive the signal transmitted from a given mobile unit and combine these multiple signals either by selecting the strongest one or summing the signals together, perhaps with some optimized weights. This typically increases SNR and reduces the effects of shadowing. Combining of signals received from multiple base stations is called macrodiversity, and here we explore the benefits of this technique. Diversity will be covered in more detail in Chapter 7. Consider a mobile at the midpoint between two base stations in a cellular network. The received signals (in dBW) from the base stations are given by

$$P_{r,1} = W + Z_1, \qquad P_{r,2} = W + Z_2,$$

where Z_1, Z_2 are $N(0, \sigma^2)$ random variables. We define outage with macrodiversity to be the event that both $P_{r,1}$ and $P_{r,2}$ fall below a threshold T.

(a) (10 pts) Interpret the terms W, Z_1, Z_2 in $P_{r,1}$ and $P_{r,2}$. If Z_1 and Z_2 are independent, show that the outage probability is given by

$$P_{out} = [Q(\Delta/\sigma)]^2,$$

where $\Delta = W - T$ is the fade margin at the mobile's location.

(b) (10 pts) Now suppose that Z_1 and Z_2 are correlated in the following way:

$$Z_1 = aY_1 + bY, \qquad Z_2 = aY_2 + bY,$$

where Y, Y_1, Y_2 are independent $N(0, \sigma^2)$ random variables and where a, b are such that $a^2 + b^2 = 1$. Show that

$$P_{out} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\Delta + by\sigma}{|a|\sigma} \right) \right]^2 e^{\frac{-y^2}{2}} dy.$$