EE 359 MIDTERM - AUTUMN 2017

The exam is open notes, open book, with no communication devices allowed (computers, cell phones, etc.). There is a plot of the Q function at the end of the exam.

1. **Adopting adaptation for airwave amelioration.** (*40 points*)

   After taking EE359, you decide to become an LTE wireless operator to compete with AT&T and Verizon in your neighborhood. After performing some measurements in your neighborhood, you learn that the channel is a discrete, time-varying AWGN channel with three states. Assuming fixed transmit power $\bar{P}$, the received SNR associated with each channel state is $\gamma_1 = 3dB, \gamma_2 = 10dB$, and $\gamma_3 = 20dB$, with probabilities $p_1 = 0.3$, $p_2 = 0.4$, and $p_3 = 0.3$, respectively. Assume that both the transmitter and receiver have perfect instantaneous estimates of the channel.

   (a) (*8 points*) Find the optimal transmission strategy and associated Shannon capacity per unit Hertz of this channel.

   (b) (*8 points*) Find the capacity per unit Hertz of Verizon's LTE deployment in your neighborhood where, in this system, the power is not adapted, only rate (i.e. transmit power is fixed at $\bar{P}$ for all transmissions).

   (c) (*8 points*) Find the capacity per unit Hertz of AT&T's LTE deployment in your neighborhood where, in this system, the fading is inverted through channel inversion power adaptation.

   (d) (*4 points*) Suppose your system has a total bandwidth of 10 MHz. How much bandwidth would Verizon and AT&T, respectively, need to purchase in order to provide the same data rate as your system?

   (e) (*4 points*) Suppose your power amplifier has a peak power constraint of $\bar{P}$. What is the optimal power and rate adaptation, and the associated capacity.

   (f) (*8 points*) Suppose your power amplifier only allows a transmit power that is 5% higher than $\bar{P}$, i.e. it has a peak power constraint of $1.05\bar{P}$. Determine the optimal power and rate allocation to maximize capacity under this new constraint.

2. **More microdiversity for Mueller's Moscow meddling mousehunt.** (*30 points*)

   Robert Mueller is planning a stakeout in downtown London to intercept communications between a former senior campaign official and a professor with alleged ties to the Russian government. He has come to you asking for help. The campaign official is transmitting using $10W$ of power and an omnidirectional antenna at 3GHz using BPSK modulation. Mueller plans to park his surveillance van 200m down the street. The receiver in the van has an AWGN noise floor of -51dBm.

   (a) (*6 points*) Using the free-space pathloss model, what antenna gain (in dB) is required at the surveillance van to ensure a minimum BER of 10^{-6} at a distance of 200m? Assume no fading or shadowing for this case.

   (b) (*8 points*) To account for losses caused by additional channel impairments, Mueller choses an antenna that has 6dB more gain than the answer you give him for part a. It is late at night and the only traffic on the street consists of large trucks intermittently passing by along with pedestrian traffic. This leads to slow log-Normal shadow fading with $\sigma = 4.2$dB. Assume that an outage occurs when the BER is less than 10^{-6}. What percentage of the communications will now be intercepted by the van?

   (c) (*8 points*) It is now rush hour and the streets are full of vehicle and pedestrian traffic. The channel now experiences Rayleigh fading and the effect of shadow fading becomes negligible. What is the outage probability in this case? How close must the surveillance van be moved so that $P_{out} \leq 0.05$? If the channel Doppler is 80 Hz, for what values of the symbol time T_s is outage probability a good performance metric?

(d) (*8 points*) Being a savvy EE359 graduate, you recommend to Mueller that, rather than moving the van closer, a diversity scheme should be installed in the van to combat fading. Using SC combining, what is the minimum number of diversity branches you need to achieve $P_{out} \le 0.01$ while leaving the van parked at 200m?

3. **Small Scale Fading Exploration.** (*30 points*)

Consider a wireless channel consisting of N components arriving at a receiver moving at velocity v. Each component has a constant random amplitude α_n, delay τ_n, and arrives at the receiver with angle θ_n with its direction of motion. The carrier frequency is f_c (with wavelength λ). The time-varying impulse response of the channel is given by

$$c(\tau, t) = \sum_{n=0}^{N} \alpha_n \delta(\tau - \tau_n) e^{-j2\pi\left[f_c \tau_n - \frac{vt\cos\theta_n}{\lambda}\right]}$$

The signal $u(t)$ with bandwidth B_u is transmitted. Assume that the expected power of each multipath component is the same, i.e. $E[\alpha_n^2 u^2(t)] = P_r/N$ where P_r is a constant. Further, suppose the receiver is very directional so that for some p_0 and p_1, $0 \le p_0, p_1 \le 1$ with $p_0 + p_1 = 1$, the incoming angles follow this distribution

$$\theta_n \overset{\text{iid}}{\sim} \begin{cases} 0 & \text{with probability } p_0/2 \\ \pi/2 & \text{with probability } p_1/2 \\ \pi & \text{with probability } p_0/2 \\ 3\pi/2 & \text{with probability } p_1/2 \end{cases}$$

(a) (*8 points*) Assume $E[\max_n \tau_n] \ll B_u^{-1}$. We know that the received signal in baseband can be written as

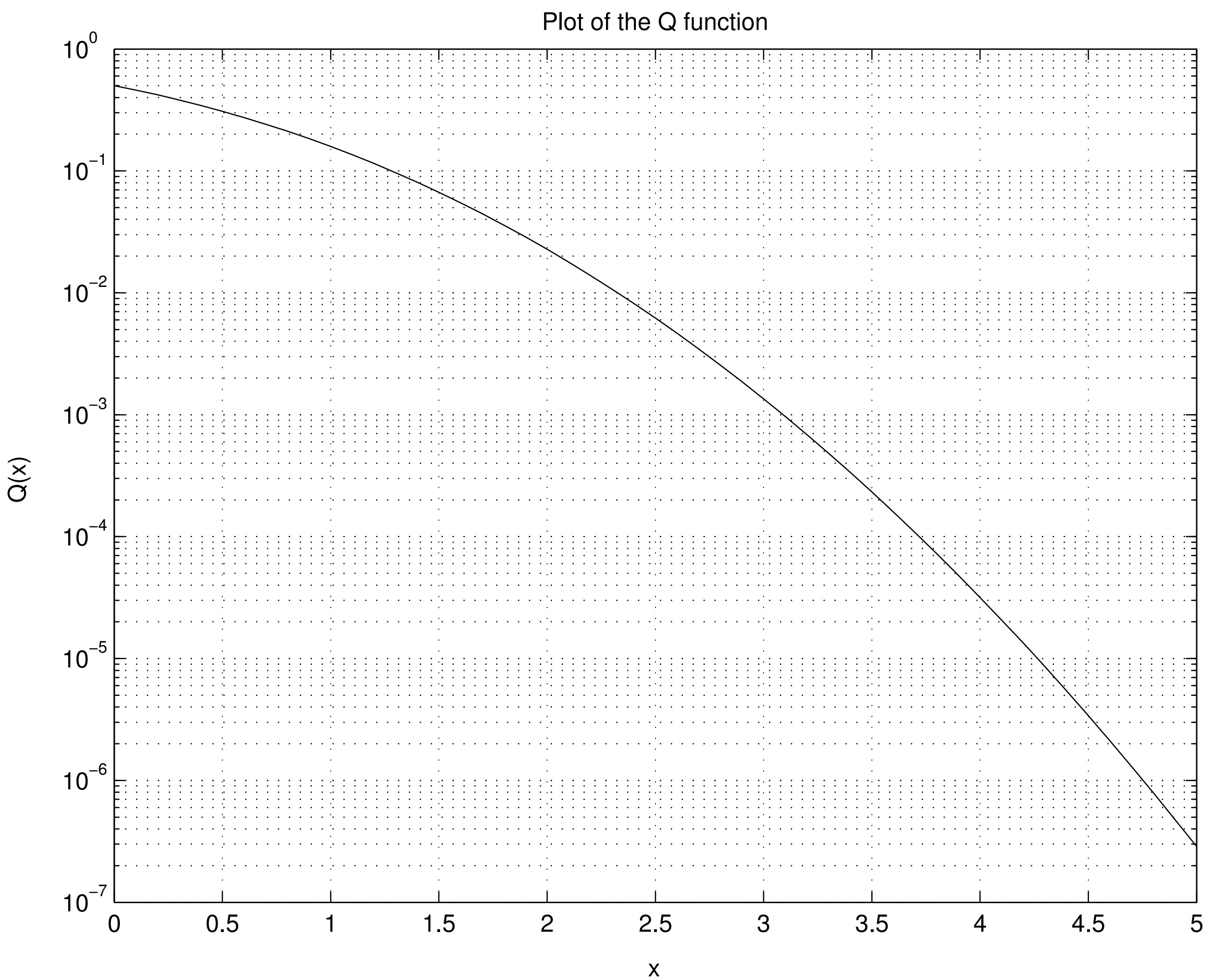
$$r(t) = r_I(t) - jr_Q(t).$$

Now, with large N, $r_I(t)$ and $r_Q(t)$ are jointly Gaussian random processes. Characterize these processes by finding the means $E[r_I(t)]$ and $E[r_Q(t)]$ and correlations $A_{r_I}(\Delta t) = E[r_I(t)r_I(t+\Delta t)]$, $A_{r_Q}(\Delta t) = E[r_Q(t)r_Q(t+\Delta t)]$ and $A_{r_I,r_Q}(\Delta t) = E[r_I(t)r_Q(t+\Delta t)]$.

(b) (*8 points*) Recall that the Doppler spread can be obtained from the support of the Doppler power spectrum $S(\Delta f) = \mathcal{F}_{\Delta t} A_{r_I}(\Delta t)$. Obtain the Doppler power spectrum and determine the Doppler spread in the cases where (i) $p_0 = 1$ and (ii) $p_1 = 1$. Which of these 2 cases corresponds to a static channel and why is the channel static in this case even when velocity v is nonzero?

(c) (*8 points*) An outage event is declared if $|r(t)|^2 \le P_r/2$. What is the outage probability if we further know that $|\alpha_n u(t)| = |z_n|$, $z_n \overset{\text{iid}}{\sim} \mathcal{N}(0, P_r/N)$? Does your answer change if the amplitudes are constant $|\alpha_n u(t)| = \sqrt{P_r/N}$?

(d) (*6 points*) Now, we no longer make the assumption that $E[\max_n \tau_n] \ll B_u^{-1}$. If the multipath delay power spectrum has uniform power c up to some delay T_{max}, i.e. $A_c(\tau) = c$, $0 \le \tau \le T_{max}$ and is zero otherwise, find the mean delay spread and rms delay spread.



Plot of the Q function
Q(x)
x
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5