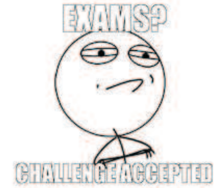


EE 359 MIDTERM - AUTUMN 2016

The exam is open notes, open book, with no communication devices allowed (computers, cell phones, etc.).

1. **Kanye's Grand Tour (30 points):**

   You scored a ticket for Kanye West's "greatest tour ever". The concert auditorium is a circular building of radius 200 m. The path loss model in the auditorium follows the simplified model with $K = .1$, $d_0 = 1$m, and path loss exponent $\gamma = 3$. There is a WiFi access point (AP) at ground level in the center of the auditorium with transmit power 60mW. The noise power in the receiver within the signal bandwidth of your WiFi signal is 10^{-10} W. Note that you can use the Chernoff bound in this problem to approximate the Q-function as $Q(x) \approx .5e^{-x^2/2}$.

   a) (7 pts) You are the first fan to arrive and start wandering around the auditorium. When you upload selfies to the AP in the empty auditorium the power falloff with distance is due to path loss alone. For DPSK modulation, what percentage of locations in the hall have $P_b \leq 10^{-4}$?

   b) (9 pts) The fans have arrived, the concert is about to begin, and you want to upload a video to Facebook. The fan bodies are causing log normal shadowing with $\sigma_{\psi_{dB}} = 4$. You are standing 100m from the AP. What is $p(P_b > 10^{-3})$ at this location?

   c) (9 pts) Now the concert is in full swing and the stage props are moving around, causing fast Rayleigh fading on top of the log-normal shadowing in part (b). For this new propagation model, find the probabiity that $\overline{P}_b \leq 10^{-2}$ under BPSK modulation at 100m from the AP.

   d) (5 pts) If the multipath delay spread in the auditorium has the random multipath delay T_m uniformly distributed between 0 and 5microsec, what is the rms delay spread and the maximum data rate of DPSK for which the multipath fading remains flat (i.e. there is negligible ISI).

2. **Green Wireless, Brad Pitt, and Bicycle Riding (35 points):** Brad Pitt is designing a wireless system for the remote village where he plans to write his Brangelina memoirs. Given how remote it is, the transmitter must be powered by riding a stationary bicycle and collecting the associated energy. This problem investigates how to achieve a given performance with minimum average transmit power (minimal bicycle exercise exertion) by Brad in order to send his memoirs to his publisher. Assume Brad's wireless channel experiences flat-fading channel with an instantaneous SNR $\gamma(t) = g(t)\overline{P}/(N_0B)$ that has pdf $p(\gamma)$, where $\sqrt{g(t)}$ is the instantaneous channel gain and $\overline{P}$ is the average transmit power.

   (a) (8 pts) For DPSK modulation with a target BER of 10^{-3}, in Rayleigh fading how much average transmit power $\overline{P}$ does Brad need to generate from bicycling for an outage probability of .01, assuming the average channel gain $\overline{g}$ is .0001 and $N_0B = 10^{-10}$ Watts.

   (b) (8 pts) How much less transmit power does Brad need to generate compared to part (a) if his receiver has three-branch selection combining?

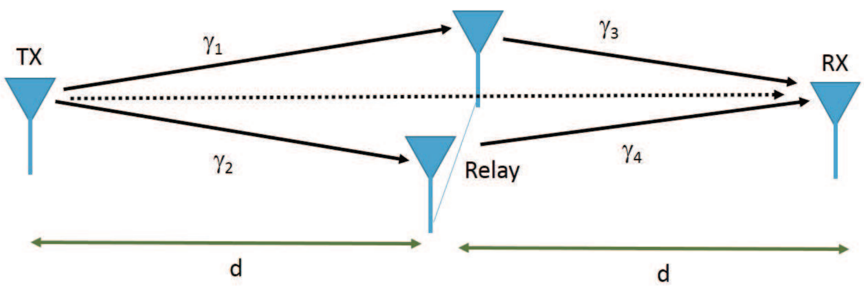
   (c) (4 pts) Is the outage metric in parts (a) and (b) most appropriate when $RT_c >> 1$ or when $RT_c \approx 1$ for R the data rate of the DPSK and T_c the coherence time of the fading?

   (d) (15 pts) Brad now wants to use Shannon's wisdom to minimize his bicycling to achieve a given rate. For a given pdf for g, $p(g)$, determine the optimal power adaptation $P(g)$ with minimum average transmit power $\overline{P}$ that achieves a given average channel capacity C_{target}, i.e. for which

   $$\int_0^\infty B \log\left(1 + \frac{gP(g)}{N_0B}\right) p(g)dg = C_{target}.$$

   Be sure to indicate how your formula for $P(g)$ depends on C_{target} and the distribution $p(g)$ (which can be any distribution).

3. **Diverse Superhero Collaboration - benefits of multi-antenna relaying (35 pts):** Consider the relay system shown in the figure below. Superman is transmitting with a single-antenna transmitter (TX), Spiderman is receiving with a single-antenna receiver (RX) a distance $2d$ from Superman, and Wonder Woman is in the middle with a two-antenna relay (Relay) that can receive using receiver diversity, and transmit using transmit diversity (recall that if the channel gains are known at the transmitter, the diversity performance of TX diversity is the same as under RX diversity with channel gains known at the receiver). The relay cannot transmit and receive at the same time (i.e. it is *half-duplex*), so the link from the TX to the relay is used a fraction τ of the time and the link from the relay to the RX is used a fraction $(1-\tau)$ of the time. As shown in the figure, the SNR associated with the channel between the TX and each relay antenna is $\gamma_i, i = 1, 2$, and between each relay antenna and the RX is $\gamma_i, i = 3, 4$. Since the relay is half-duplex, there is only relaying via Wonder Woman, or only direct transmission between Superman and Spiderman (shown by the dashed line in the figure), i.e. both the relay and direct transmission cannot be used simultaneously. Assume the signal bandwidth of this system is $B = 10$ MHz and that the separation distance between the relays is very small compared to d.



   (a) (5 pts) Let C_1 denote the capacity of the channel from the TX to the relay, and C_2 denote the capacity of the channel from the relay to the RX. Explain why the capacity C_{e2e} of the end-to-end system from the TX to the RX is $\min(\tau C_1, (1-\tau)C_2)$, assuming only the relay is used. Given C_1 and C_2, what value of τ maximizes this capacity?

   (b) (10 pts) Consider that the γ_is are based only on the simplified path loss model (i.e. the channel is AWGN with path loss only), with $K = 1$ and $d_0 = 1$m, $d = 10$m, path loss exponent equal to two, and $P_t/(N_0 B) = 30$dB where P_t is the transmit power at both the TX and the relay, and $N_0 B$ is the noise at all receive antennas. Assuming MRC transmit and receive diversity at the relay, find C_1, C_2, the total capacity C_{e2e} and the value of τ that maximizes this total capacity. Compare with the capacity assuming only direct transmission between the TX and RX, i.e. not using the relay.

   (c) (8 pts) Using the same model and parameters as in part (b), find a distance $d \geq d_0$ for which capacity is higher using direct transmission between the TX and RX rather than using the relay.

   (d) (12 pts) Assume now that the γ_i (in linear units) associated with the relay experience i.i.d. flat-fading, with $\gamma_i = 3$ with probability .6, and $\gamma_i = 10$ with probability .4. Assume that the channel gains are known to the TX, RX and relay and that the transmit power at both the TX and relay are fixed (i.e. there is no power adaptation). If the relay uses MRC receive diversity and SC transmit diversity, find C_1, C_2, C_{e2e} and the value of τ that maximizes this total capacity.

The Shannon capacity of this relay channel assumes full-duplex transmission (the relay can transmit and receive at the same time), and hence it also assumes that the relay and direct transmission can be used simultaneously. The Shannon capacity of the relay channel under these assumptions has been an open problem for decades. Up until a few years ago, full-duplex transmission was not used in practice, but recently technology has evolved such that full-duplex is commercially viable, with full-duplex startup companies like Kumu Wireless currently in field trials with cellular operators.