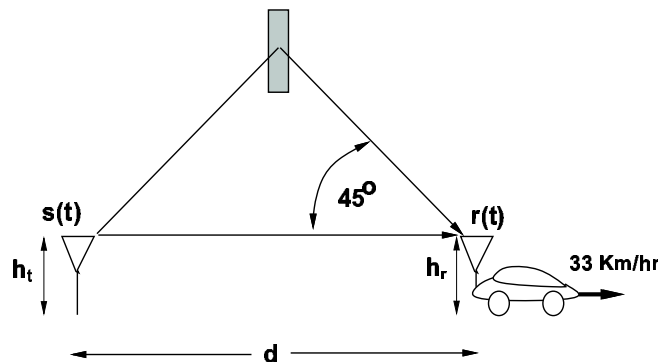


# EE 359 MIDTERM - AUTUMN 2015

The exam is open notes, open book, with no communication devices allowed (computers, cell phones, etc.). You may use any formulas derived in class or in the book as long as you reference them. Good luck.

## 1. Donald Trump's Poll Numbers (30 points)

Donald is driving through Manhattan monitoring his poll numbers. The wireless channel he experiences is shown in the figure below, which consists of a LOS path and one multipath component (bouncing off the Trump Tower) for each distance  $d$  between the transmitter and receiver. In particular, for every distance  $d$ , the Trump Tower gives rise to a single multipath component at an arrival angle of 45 degrees (the figure shows the reflector giving rise to the multipath component for a given distance  $d$ ). Assume Donald is traveling at 33 Km/hr in a straight line directly away from the transmitter, the transmitted power is 10W, and the carrier frequency is 1 GHz. Also assume the antennas at the transmitter and receiver are separated by distance  $d = 4$  Km and that these antennas are nondirectional (0 dB antenna gain) and at the same height above the ground,  $h_r = h_t = 6$  m.



- Sketch the scattering function  $S_c(\tau, \rho)$  at the snapshot in time associated with the figure. Be sure to label all axes and label all relevant numerical values.
- What is the channel coherence time and coherence bandwidth?
- Is the distribution of the received signal envelope Rayleigh, Ricean, Nakagami or none of these? Explain.
- Suppose now that Donald is driving through a dense forest area of Central Park at 33 Km/hr, so that his received signal experiences uniform scattering and Rayleigh fading. Assume his maximum delay spread is the same as in the above figure. For what range of data rates can a DPSK-modulated signal be sent over this channel with error probability less than  $10^{-3}$ . What received SNR is required to obtain this error probability?



## 2. Tesla Fever (35 points)

You are the proud owner of a new Tesla, and want to test the capacity limits of your “connected car”.

- (a) Tesla’s cheap transmitter amplifiers impose a fixed peak power constraint in addition to an average power constraint on the wireless transmitter. What is the optimal power and rate adaptation of a flat-fading channel to maximize average capacity, assuming both an average power constraint  $\bar{P}$  and a peak power constraint  $P_{pk} > \bar{P}$ . Sketch the modification of Figure 4.5 in the text to show how water-filling power allocation must be modified due to this peak constraint. Explain the qualitative effect on capacity of a fixed peak power constraint when  $P_{pk} \gg \bar{P}$  and when  $P_{pk} \approx \bar{P}$ .

For the remainder of the problem, consider that you are driving through a circular cell of radius 1Km. Assume the transmit power at the base station is .7 W and that path loss from the base to mobiles in the cell follows the simplified model  $P_r = P_t K (d/d_0)^{-3}$  with  $K = 1$  and  $d_0 = 1$  m. Assume also a signal bandwidth of  $B = 100$  KHz and that the receiver noise power spectral density is  $N_0 = 10^{-15}$  W/Hz.

- (b) Find the capacity of your Tesla’s wireless system under channel inversion (i.e. the zero-outage capacity) when you are driving along a radial line from distance  $d_0$  from the base station to the cell boundary, assuming you are equally likely to be anywhere on the line.
- (c) Suppose there is a tunnel along one quarter of the periphery of the cell. The tunnel causes a 5.44 dB signal attenuation. Find the Shannon capacity of your Tesla’s wireless system assuming you are equally likely to be anywhere along the cell boundary under perfect transmitter and receiver channel side information **and** under perfect receiver channel side information only.



*Only one more problem to go*

## 3. Rayleigh and Nakagami: Pessimist versus Optimist (35 points)

The problem investigates which person came up with a more pessimistic channel model: Rayleigh or Nakagami. It also illuminates a connection between BER performance of single-branch Nakagami- $m$  fading and  $m$ -branch MRC combining of iid Rayleigh fading branches. Consider a DPSK system with  $N$ -branch receiver diversity, MRC combining, and an average SNR of  $\bar{\gamma}$  on each branch. Note that this system is best analyzed using the MGF approach.

- (a) Assuming iid Rayleigh fading, for  $\bar{\gamma} = 20$  dB, how many diversity branches are needed to achieve a target average BER of at most  $10^{-6}$ ?
- (b) Assuming Nakagami- $m$  fading with  $m = 3$ , for  $\bar{\gamma} = 20$  dB, how many diversity branches  $N$  are needed to achieve a target average BER of at most  $10^{-6}$ ? Comparing with your answer to part (a), why do you need fewer branches?
- (c) Assume again iid Rayleigh fading with 3-branch MRC diversity,  $\bar{\gamma} = 20$  dB, and an additional shadowing term of  $\psi$  that is the same for all branches, so that the received SNR on each branch is  $\gamma\psi$ . If  $\psi$  is uniformly distributed between zero and one, what is the probability that the average BER due to Rayleigh fading will exceed  $10^{-3}$ ?
- (d) For an  $N$ -branch MRC diversity system with Nakagami- $m$  fading, what is the diversity order of the system (change in slope of the BER curve) in the limit of high SNR.