EE 359 - Wireless Communications - Autumn 2017

Final: Total 200 Points, Duration: 12:15pm-3:15pm

The exam is open book and notes. Please state all assumptions used in your calculations. You may use any derivations or statements from the book as long as you cite where they come from.

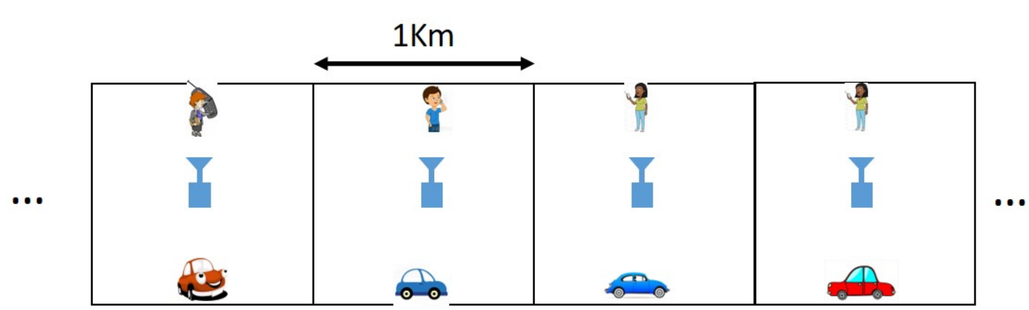
1. Short answer questions

   (a) **Non-coherent systems:** (8 pts). Consider a communication system where the receiver cannot obtain phase information, only amplitude information. Hence the received signal is $y = |5x| + n$ where x is the transmit symbol and n is iid Gaussian noise of power 1. What is the error probability of BPSK modulation used on this channel for equiprobable signaling? Assume an amplitude-only modulation scheme is used where a 0 bit corresponds to sending zero energy over a symbol time and a 1 bit corresponds to sending energy A over a symbol time. If for equiprobable signaling, this scheme has an average power 1, find the probability of error. You can assume $Q(x) \approx (1/(\sqrt{2\pi(1+x^2)}) \cdot \exp\left(-x^2/2\right)$.

   (b) **Performance of RAKE Receivers:** (12 pts) Consider a 2-branch RAKE receiver. The fading distribution on each branch is $p(\gamma)$ where $\int_0^\infty p(\gamma)e^{x\gamma}d\gamma = .01\overline{\gamma}/\sqrt{x}$. Find the average P_b for a spread-spectrum BPSK signal where the RAKE receiver has 2-branch diversity with MRC combining, and each branch has an average SNR of 10 dB and experiences independent fading with distribution $p(\gamma)$. (Note: $\sin^2 x = .5(1-\cos(2x))$.

   (c) **DSSS Multiuser System:** (12 pts) Consider the uplink of a two-user DSSS system. For a channel gain of unity, the received signal power of each user is 10dBm. If the DSSS code cross correlation is 0.1, what is the SINR at the receiver for each user when they have the same channel gain, if you assume the noise power is negligible relative to the interference power. What is the SINR at the receiver for each user when User 1 has a channel gain that is 10 times larger than that of User 2, again assuming that the noise is negligible?

   (d) **Cellular System Capacity:** (18 pts) Consider the one-dimensional linear cellular system shown below with an infinite set of square cells each of length 1Km with a base station (BS) in the middle of each cell and users at the top and bottom of each cell in vertical alignment with the BS. All BSs transmit over an omni-directional antenna (equal gain in all directions) and propagation follows the free space path loss formula. The BSs transmit at a power of 1W and a carrier frequency of 1 GHz, receivers have a noise power of 10^{-9} W (-90 dBW), and the system has a total bandwidth of 20 MHz. Find the total multiuser capacity in each cell (sum of Shannon capacity to each user) assuming interference is treated as noise for a reuse distance of 1 (20 MHz used in each cell) and a reuse distance of 2 (10 MHz used in alternating cells) and frequency division used within each cell as the multiple access technique. You can ignore interference coming from more than 3 Km away. What is the reuse distance that maximizes this multiuser capacity?



2. **Molecular Communications** Two startups are developing communication systems based on embedding information bits into the transmission of molecules rather than the transmission of EM waves. These systems are compelling for small scale systems such as in-body communication as well as other scenarios where EM transmission is impractical or doesn't work well (e.g. underwater). In a molecular communication system, the transmitter sends molecules, a channel (eg. our blood stream) carries these molecules typically via diffusion and flow processes, and a receiver detects the molecules. This problem explores the brilliant ideas behind the technology of these two startups.

   (a) (10 pts) Startup MakeMeRich builds a molecular system that modulates information in molecular communications in the concentration of the molecules. In this case, at a given time T_0, in order to transmit bit 0, the transmitter does not transmit anything and, for bit 1, the transmitter transmits M molecules simultaneously. The receiver detects molecules received over a given detection window T_W, so if $T_0 = 0$, the detection window spans the timeframe $[0, T_W]$. Assume that the probability that the receiver detects any one of the M molecules over the timeframe $[0, T_W]$ is ρ and that detection of each molecule is independent. Suppose that the receiver declares bit 1 if it receives any of the M molecules. Find the probability of error given that either a 0 or a 1 bit could be transmitted with equal probability.

   (b) (15 pts) Startup NextBigThing modulates information in molecules based on timing. Again assume the transmission window begins at $T_0 = 0$. Suppose that the transmitter releases a particle at time $t = 0$ for bit 0 and at $t = T$ for bit 1. The receiver decodes over the timeframe $[0, T_W]$ for $T_W \gg T$ by declaring bit 0 if it detects the particle before time $t = L$ and bit 1 if it detects the particular over the timeframe $[L, T_W]$. Assuming the distribution of travel time is exponentially distributed with mean λ, find an expression for the probability of error under equiprobable signaling as a function of λ, L, T, and T_W. Based on your expression, show that $T = L$ minimizes this probability of error.

3. **OFDM.** Trump is talking with Jeff Sessions about the HUGE Alabama special election outcome over a channel with the following channel autocorrelation function:

$$A_c(\tau, \Delta t) = 10^5 \cdot \frac{\sin(600\pi\Delta t)}{\pi\Delta t} \cdot \exp\left(-10^6\tau\right).$$

   (a) (8 pts) What is approximate coherence bandwidth and Doppler spread of this channel?

   (b) (12 pts) For a system bandwidth of 40MHz, find the required number of OFDM subcarriers to ensure that each subchannel approximately experiences fading that is approximately flat; the number of subcarriers must be a power of two so that the FFT can be implemented in $O(N \log N)$ operations. Additionally, find the length of the cyclic prefix so that ISI is approximately zero between OFDM blocks. Please use average delay spread rather than maximum delay spread for this computation.

   (c) (18 pts) For the subchannels in part (3b), suppose power is allocated equally across all subchannels and that each subchannel can be modeled as an AWGN channel with an SNR of 20 dB (20 dBm TX power and 0 dBm of noise). As Trump is moving slowly, for now ignore the effects of Doppler (that is, assume the autocorrelation function is $A_c(\tau, \Delta t) = e^{-10^6\tau}$). What is the maximum data rate at which Trump can send data to Sessions by this system using MQAM modulation with no restrictions on constellation size and a BER target of 10^{-6}?

   (d) (12 pts) One effect of Doppler spread on an OFDM system is intercarrier interference (ICI). Each subcarrier will be spread in frequency causing energy to spill over into adjacent subcarriers. In this system, since the subcarrier bandwidth is large compared to the Doppler spread, we can use the rule-of-thumb approximation that 98% of the original subcarrier power will remain in the subcarrier bandwidth and the remaining 2% will spill into the neighboring subcarriers. Treating ICI as noise, what is the loss of the data rate computed in part (c) when the degradation due to ICI is taken into account?

4. **Spatial Multiple Access.** The multiple spatial dimensions of MIMO systems also allows multiple users to occupy independent spatial dimensions so they can coexist without interfering with each other. This problem explores the design and performance of multiuser MIMO systems.

Jay-Z wishes to send a message to Beyoncé over a 4×4 MIMO system. The channel gain matrix between them:

$$\mathbf{H}_{jb} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{H}}$$
$$= \begin{bmatrix} -0.5678 & 0.5856 & 0.1348 & -0.5627 \\ -0.5438 & -0.3211 & -0.7748 & 0.0290 \\ 0.3515 & 0.7203 & -0.5352 & 0.2667 \\ -0.5083 & 0.1876 & 0.3083 & 0.7819 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.8 & 0 & 0 \\ 0 & 0 & 0.75 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -0.4149 & -0.8068 & 0.0883 & -0.4113 \\ 0.0333 & 0.1663 & 0.9739 & -0.1508 \\ -0.9092 & 0.3725 & -0.0038 & 0.1857 \\ -0.0036 & 0.4274 & -0.2090 & -0.8796 \end{bmatrix}.$$

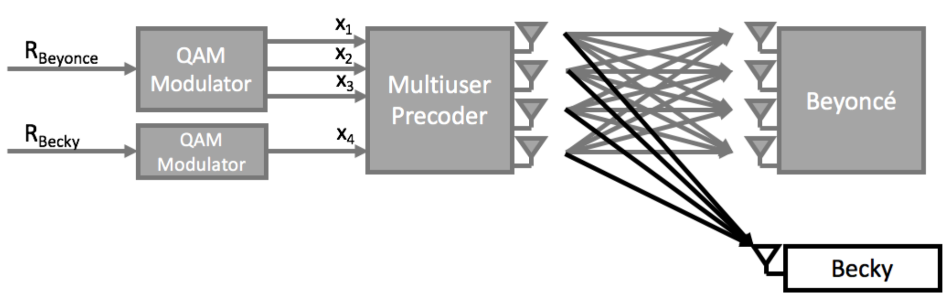
Assume a system bandwidth of $B = 1$ MHz, a noise power level of 0 dBm at Beyoncé's location, and a transmit power level of 20 dBm at Jay-Z's location. Further assume that both users have perfect CSI knowledge.

(a) (10 pts) What is the rank of $\mathbf{H}$ and the SNR γ_i associated with each of its spatial dimension assuming the full transmit power is allocated to it?

(b) (20 pts) What is the capacity of this channel? How is this capacity acheived?

(c) (15 pts) Using the capacity achieving power allocation (that found in part (4b), what is the largest size MQAM constellation supported along each spatial dimension? Assume Jay-Z is restricted to using the constellations $M = \{2, 4, 16, 64, 256\}$.

While communicating with Beyoncé, Jay-Z wants to also send a message to Becky with the Good Hair. Becky with the Good Hair is not a celebrity so she can only afford a single (high-gain) antenna. She has a noise floor of 0 dBm and shares the same 1 MHz of bandwidth with Beyoncé.



(d) (10 pts) In order to send data to both users simultaneously, Jay-Z will make use of a multiuser precoding matrix. The multiuser precoding matrix is a 4×4 matrix $\mathbf{P}$, which takes as input symbols $x_1, \ldots, x_4$ and outputs $\tilde{\mathbf{x}} = \mathbf{P}\mathbf{x}$. Jay-Z will dedicate symbols x_1, x_2, and x_3 to sending to Beyoncé and symbol x_4 to sending to Becky. When Jay-Z is only communicating to Beyoncé he sends $x_4 = 0$. What single-user precoding matrix should Jay-Z use in this case?

(e) (10 pts) In order to send data to Becky with the Good Hair, Jay-Z chooses the fourth column of $\mathbf{P}$ to be the fourth right singular vector of the channel between him and Beyoncé. In other words, he will precode with the first three columns of $\mathbf{P}$ as found in part (4d) and the fourth column of $\mathbf{P}$ as $\mathbf{v}_4 = [-0.0036; 0.4274; -0.2090; -0.8796]$. The average power of symbols x_1, x_2, x_3 will remain 20 dBm and he will use an additional 20 dBm on symbol x_4. What fraction of the 20 dBm power in x_4 will Beyoncé receive?

(f) (10 pts) The channel gains between Jay-Z's 4 transmit antennas and Becky with the Good Hair's single receive antenna are $h_{jb_{wtgh}} = [-3.2, 2.4, -1.2, -3]$. What is Becky's received signal strength (not SNR, i.e. ignore noise and interference)?