EE 359 - Wireless Communications - Autumn 2016

Final: Total 200 Points, Duration: 12:15pm-3:15pm

The exam is open book and notes. Please state all assumptions used in your calculations. You may use any derivations or statements from the book as long as you cite where they come from.

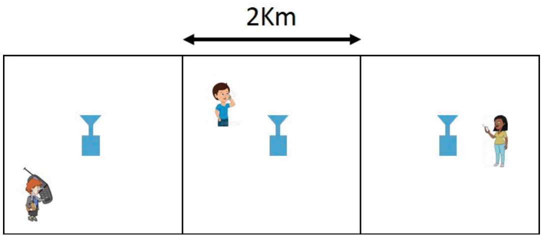
1. **Problem 1 (50 points): Short Answer**

   (a) **Discrete-rate vs. Continuous rate adaptative modulation:** (15 pts) Consider a discrete rate adaptive modulation system with continuous power adaptation to meet the BER target. Thus the region boundaries are set according to the formula $M(\gamma) = \gamma/\gamma_K^*$, i.e. the boundaries as given as $\gamma_j = \gamma_K^* M_j$ for each possible constellation size M_j. Is the γ_K^* that maximizes the spectral efficiency of this system bigger, smaller, or the same as the γ_K that satisfies the average power constraint in continous rate adaptation, and why? If we set $\gamma_K^* = \gamma_K$, which system has a higher spectral efficiency and why? For $\gamma_K^* = \gamma_K$ and the same target instantaneous BER for both systems, which system requires a higher average transmit power and why?

   (b) **OFDM design:** (15 pts) Consider a channel with maximum delay spread $T_m = 2\mu$ sec. Is this likely to be an indoor or outdoor channel and why? Suppose you build an OFDM system for this channel with a total system bandwidth of 64 MHz. Assuming nonoverlapping subchannels, how many subchannels are needed to obtain approximately flat fading on each subchannel? How many samples long does the cyclic prefix need to be to insure no ISI between FFT blocks, assuming the subchannel symbol time equals the inverse of the subchannel bandwidth?

   (c) **MIMO receiver design:** (10 pts) Consider both a sphere decoder and an ML decoder for a MIMO system. Given the decoder input $\mathbf{y}$, the sphere decoder outputs symbol $\hat{\mathbf{x}}(\mathbf{y})_{SD}$ while the ML decoder outputs $\hat{\mathbf{x}}(\mathbf{y})_{ML}$, both of which are functions of the known channel matrix $\mathbf{H}$. Suppose for a given sphere decoder radius r there are exactly two points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the transmitted signal constellation that satisfy $||\mathbf{y} - \mathbf{H}\mathbf{x}_i|| < r, i = 1, 2$. Under what conditions on $\mathbf{H}\mathbf{x}_1$ and $\mathbf{H}\mathbf{x}_2$ will $\hat{\mathbf{x}}(\mathbf{y})_{SD} = \mathbf{x}_1$? Under these conditions, will $\mathbf{x}_1 = \hat{\mathbf{x}}(\mathbf{y})_{ML}$? For what values of r is $\hat{\mathbf{x}}(\mathbf{y})_{SD} = \hat{\mathbf{x}}(\mathbf{y})_{ML}$ never true and for what values of r is it always true?

   (d) **Cellular system SIR:** (10 pts) Consider the one-dimensional linear cellular system shown below with 3 square cells (a left cell, middle cell, and right cell) each of length 2Km with a base station in the middle of the cell. All base stations and mobiles transmit over an omni-directional antenna (equal gain in all directions). Assume signal propagation follows the free space path loss formula, that the same channel is used in every cell, that all mobiles transmit at a power of P_m and all base stations transmit at a power P_b. Neglecting noise and assuming each cell has exactly one mobile, find for the uplink the worst-case locations of the interfering users in the left and right cells as well as the worst-case location of the user in the middle cell that results in the worst-case SIR for that middle user at its base station. For this same location of all 3 users, find the downlink SIR to the user in the middle cell.



2. **Problem 2 (50 Points): MIMO** Consider a 3 × 3 MIMO system with the following channel matrix

$$\mathbf{H} = \begin{bmatrix} .1 & -.3 & .5 \\ -.1 & .2 & .4 \\ -.2 & -.3 & .3 \end{bmatrix}$$

This can be written via the SVD as

$$\mathbf{H} = \begin{bmatrix} 0.7461 & -0.2241 & -0.6271 \\ 0.3827 & 0.9149 & 0.1284 \\ 0.5449 & -0.3357 & 0.7683 \end{bmatrix} \begin{bmatrix} .75985 & 0 & 0 \\ 0 & .38576 & 0 \\ 0 & 0 & .23199 \end{bmatrix} \begin{bmatrix} -0.0956 & -0.4090 & 0.9075 \\ -0.1212 & 0.9097 & 0.3972 \\ -0.9880 & -0.0720 & -0.1365 \end{bmatrix}$$

Assume the total transmit power is 12 dBm, the noise power at each receive antenna within the signal bandwidth is 0 dBm, and the system bandwidth is $B = 10$ MHz. Assume throughout the problem that modulation is unrestricted MQAM and use the BER approximation in AWGN that BER $\approx .2e^{-1.5\gamma/(M-1)}$ for your calculations.

(a) (20 pts) Find the capacity of this channel assuming optimal power and rate adaptation and under beamforming. Will the difference between these two capacities increase or decrease as the total transmit power decreases, and why?

(b) (15 pts) For a target BER of 10^{-4} and power equally divided among all spatial streams utilized, find the total maximum data rate associated with all data streams that can be sent over the channel for beamforming (1 spatial stream), using 2 out of the 3 spatial streams (also called precoding), and spatial multiplexing (3 spatial streams), assuming that $T_s = 1/B$.

(c) (15 pts) Assume now that the system experiences Rayleigh fading so that the squares of the singular values of the 3x3 MIMO system become exponentially distributed. Assume the average SNR on the ith spatial dimension of the channel is 30, 15 dB, and 10 dB. Under adaptive beamforming that selects the spatial dimension with the largest instantaneous SNR, find the probability that the instantaneous BER of DPSK modulation is above 10^{-3}.

3. **Problem 3 (50 pts): ISI Mitigation**

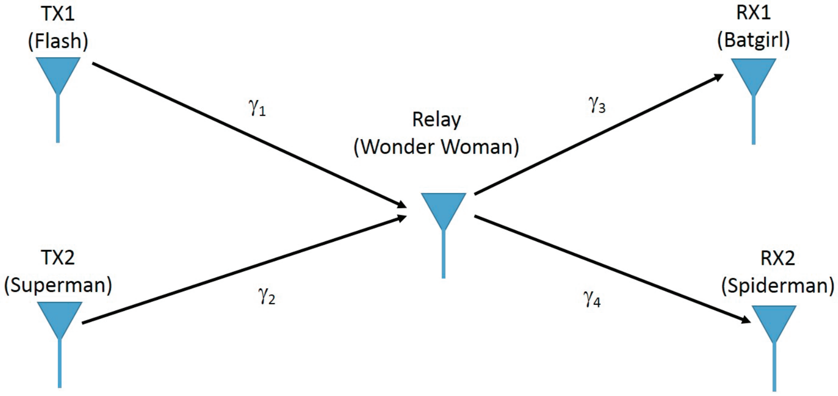
(a) (10 pts) Consider an indoor wireless system supporting low-rate data for low-cost low-power devices, e.g. the Internet of Things. Considering that low-cost low-power electronics typically have non-linear power amplifiers and run off small batteries, would you choose equalization or OFDM as your ISI mitigation technique for this device and why? Why might your decision change if this was for an outdoor cellular system with cells that covered a large part of a city?

(b) (20 pts) Consider an OFDM system with 8 AWGN subchannels of total system bandwidth 10 MHz, where the SNR on each subchannel for a transmit power per subchannel of 10 mW is $\gamma_i = 10/i, i = 1, ..., 8$. For a total transmit power of 80mW, compare the capacity of this system when the total transmit power is allocated equally over all subchannels versus when the gain on each channel is inverted such that each channel has the same received SNR.

(c) (20 pts) Consider now a DSSS system with spreading codes such that the spreading code autocorrelation is zero for $\tau > 1\mu$s. For a 4-path channel with propagation delays $\tau_0 = 1\mu$sec, $\tau_1 = 1.8\mu$sec, $\tau_2 = 2.5\mu$sec and $\tau_3 = 3.3\mu$sec, assuming all paths had the same average receiver power, if you could only build a 2-branch RAKE receiver, which multipath components would you synchronize to and why? Assume now a 2-path channel with $\tau_0 = 1\mu$sec and $\tau_1 = 3.3\mu$sec. Assume both paths have an average SNR of 10 dB and experience Rayleigh fading. Find the outage probability of DPSK modulation at a target BER of 10^{-3} for a CDMA system when the system has no RAKE receiver and locks only to the LOS path and when the system includes a 2-branch RAKE receiver that uses MRC to combine both the LOS and the reflected path.

4. **Problem 4 (50 pts): Multiuser Systems with a Relay**

Consider the multiuser system shown in the figure below. Each transmitter transmits a signal to its receiver through the relay, which is half-duplex, so it cannot transmit and receiver at the same time. Instead, it receives from the transmitters a fraction τ of the time and transmits to the receivers a fraction $(1-\tau)$ of the time. The receivers are sufficiently far away from the transmitters such that they cannot detect the signals transmitted by TXi, $i = 1, 2$, hence the only signal they detect is from the relay. Each link (from transmitter to relay or from relay to receiver) has a static SNR (no fading, only AWGN) so the capacity on each of the links assuming they use the entire system bandwidth of $B = 10$ MHz is given by $C_i = B\log_2(1+\gamma_i)$ for γ_i the SNR on the link as shown in the figure. The total end-to-end rate that TXi sends to its receiver RXi is denoted by $C_{e2e}(i)$.



(a) (10 pts) Suppose the system uses time-division to share the uplink (TX-relay) channel, allocating a time-fraction τ_U to user 1 and $(1-\tau_U)$ to user 2. Similarly, it uses time-division to share the downlink (relay-RX) channel, allocating a time-fraction τ_D to user 1 and $(1-\tau_D)$ to user 2. For $\gamma_i = 10$dB, $i = 1, 2, 3, 4$, why must we set $\tau_U = \tau_D = .5$ to maximize the end-to-end rate that both users can get simultaneously: $C_{e2e}(1) = C_{e2e}(2)$. Find this equal end-to-end rate for each TX-RX pair and the value of τ that the half-duplex relay uses in this case.

(b) (15 pts) Repeat part (a) assuming $\gamma_1 = 10$dB, $\gamma_2 = 15$dB, $\gamma_3 = 10$dB, $\gamma_4 = 15$dB, and $\tau = .5$, i.e. find the maximum equal end-to-end rate point $C_{e2e}(1) = C_{e2e}(2)$ and the values of τ_U and τ_D that achieves this.

(c) (10 pts) Show that your answer to part (b) for $C_{e2e}(1) = C_{e2e}(2)$ would not change if the system uses frequency-division to share the uplink and downlink channels rather than time-division.

(d) (15 pts) Assume now that the system uses semi-orthogonal spread spectrum codes for multiple access on both the uplink and the downlink. Assume that the interference power is reduced by the spreading gain G of the spreading code (which is roughly the number of chips per symbol or, equivalently, the ratio of the spread signal bandwidth divided by the original signal bandwidth). Assuming a transmit power at the transmitters and relay such that $\gamma_i = 10$dB, $i = 1, 2, 3, 4$, what is the minimum spreading gain such that the spread spectrum system achieves a higher end-to-end rate for each user than in part (a).