EE359 Final Review

March 11, 2020

Announcement

Please attempt all questions even if you don't know how to get to the final answer

Will likely use Canvas to distribute and collect exams

Outline

1. Review
   - Channel models
   - Performance analysis
   - Combating fading using diversity
   - Combating fading using adaptive modulation and power
   - Point to point MIMO systems
   - Combating multipath/ISI/small B_c

2. Sample finals/discussion

Broad Topics in Course

- Channel models
  - Path loss
  - Shadowing
  - Fading
- Performance analysis
  - Capacity
  - Probability of outage
  - Probability of bit/symbol error
- Combating fading using diversity
- Combating fading using adaptive modulation and power
- Point to point MIMO systems
  - Capacity and parallel channel decomposition
  - Beamforming
  - Diversity multiplexing tradeoff
  - MIMO receivers
- Combating multipath/ISI/small B_c
  - Multicarrier modulation
  - Spread spectrum (has other uses too)

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Path loss models

Models attenuation caused by "spread" of EM waves due to finite extent of transmitter

- Free space
- 2-ray and n-ray models
- Simplified path loss models

$$P_r = P_t K \left(\frac{d_0}{d}\right)^{\gamma}$$

Valid in the far field, i.e. when d is large, γ is path loss exponent, K can depend on carrier frequency

Shadowing

Models attenuation caused by EM waves passing through randomly located objects

- Log normal shadowing assumes

$$10 \log_{10}(P_r) = 10 \log_{10}(\bar{P}_r) + S,$$

where $S \sim \mathcal{N}(0, \sigma^2_{\psi_{dB}})$ or equivalently

$$P_r(\text{dB}) = \bar{P}_r(\text{dB}) + S$$

- S is associated with location, closely located points will have correlated S (can talk of decorrelation distance X_c)

Fading

Models attenuation due to EM waves combining with random phases due to multipath

Recall: Narrowband versus wideband

- Received signal $\text{Re}\{\sum_{n=1}^{N} a_n(t) e^{-j\phi_n(t)} u[\tau - \tau_n(t)] e^{j2\pi f_c t}\}$
- Narrowband approximation $u(t) \approx u(t - \tau_n(t))$, i.e. received signal is

$$r(t) = \text{Re}\{\alpha(t) u(t) e^{j2\pi f_c t}\}$$

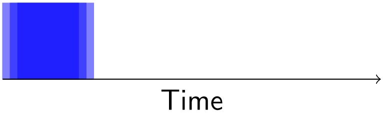


Figure: Narrowband $T_m \ll \frac{1}{B_u}$

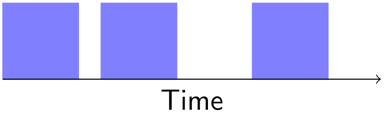


Figure: Wideband $T_m \approx, \geq \frac{1}{B_u}$

Fading contd.

Narrowband fading

- Effect of channel is just scalar multiplication by complex constant

$$\alpha(t) = r_I(t) + jr_Q(t)$$

- Specify distribution on envelope $z(t) = |\alpha(t)| = \sqrt{r_I(t)^2 + r_Q(t)^2}$: Rayleigh, Rician, Nakagami $m, \ldots$

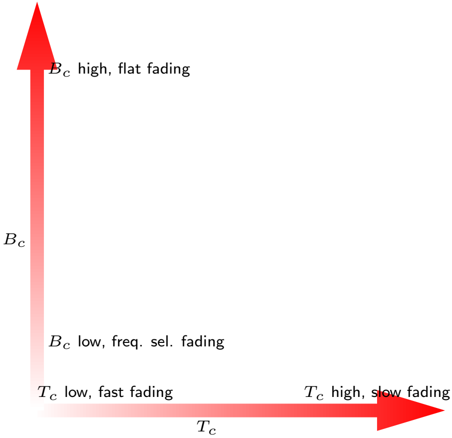
Wideband fading

- Effect of channel no longer modelled by a single scalar multiplication
- Divide up wide band into M narrow bands $(1, \ldots, m, \ldots, M)$ with fading $\alpha_m(t)$
- Specify joint distributions on $\alpha_m(t)$ for $m \in \{1, \ldots, M\}$

On fading "types"

Depends on:

- Signal Bandwidth B_u
- Coherence Time T_c or Doppler Effects
- Coherence Bandwidth B_c or Delay Spread



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Capacity

Definition

Maximum data rate that can be supported by the channel with vanishing probability of error

Capacity C under different models (γ is the instantaneous SNR at the receiver, B is bandwidth)

Scheme	**Capacity Expression**
AWGN	$C = B\log_2(1+\gamma)$
Shannon capacity in fading with Rx CSI only	$C = \int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma$
Shannon capacity in fading with constant Tx power and Tx, Rx CSI	$C = \int_0^\infty B\log_2(1+\gamma)p(\gamma)d\gamma$
Shannon capacity with Tx, Rx CSI (Waterfilling)	$C = \int_{\gamma_0}^\infty B\log_2(\gamma/\gamma_0)p(\gamma)d\gamma$, where $\int_{\gamma_0}^\infty (1/\gamma_0 - 1/\gamma)p(\gamma)d\gamma = 1$

Capacity formulas continued ...

Capacity expressions C

Scheme	**Capacity Expression**
Channel Inversion	$C = B\log_2\left(1+\frac{1}{\mathsf{E}[1/\gamma]}\right)$
Truncated Channel Inversion	$C = B\log_2\left(1+\frac{1}{\mathsf{E}_{\gamma_0}[1/\gamma]}\right)p(\gamma>\gamma_0)$ where $\mathsf{E}_{\gamma_0}[1/\gamma] = \int_{\gamma_0}^{\infty}\frac{1}{\gamma}p(\gamma)d\gamma$

Outage probability

Idea

Outage $\equiv$ Received SNR γ is below threshold γ_0

Reasons

- Path Loss (usually no randomness)
- Shadowing (randomness if shadowing time scales are small)
- Fading (randomness due to multipath combining)

Outage probability

Idea

Outage $\equiv$ Instantaneous probability of error P_e is greater than $P_{e,0}$

Reasons

- Path Loss (usually no randomness)
- Shadowing (randomness if shadowing time scales are small)
- Fading (randomness due to multipath combining)

Outage probability and cell coverage area

Outage probability

- Defined *for a particular location*
- Relates P_{out}, P_{min} (dB), $\bar{P}_r(d)$ (dB), $\sigma_{\psi_{dB}}$ at a location d via

$$P_{\text{out}} = Q\left(\frac{\bar{P}_r(d) - P_{\text{min}}}{\sigma_{\psi_{dB}}}\right) \text{ under log normal shadowing}$$

Average probability of bit/symbol error

Idea

- Compute $\bar{P}_s = \mathsf{E}_\gamma[P_s(\gamma)]$
- May be simplified using alternate Q functions and MGFs of fading distributions

Regime of relevance

Metric	**Relevant regime**
Outage probability	$T_s \ll T_c$
Average probability of error	$T_s \approx T_c$
AWGN probability of error	$T_s \gg T_c$

Error floors

What is an error floor?

Error floor whenever $P_s \nrightarrow 0$ as $\gamma \to \infty$

Summary of effects

- Data rate cannot be too low with non coherent schemes
  - Non coherent schemes assume channel is constant across subsequent symbols
  - Depends on e.g. Doppler or T_c
- Data rate cannot be too high in any system
  - Channel will "spread" symbols across time, causing self interference (ISI — inter symbol interference)
  - Depends on e.g. B_c or coherence bandwidth of channel

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Diversity

Idea

Use of independent fading realizations can reduce the probability of error/outage events

Some diversity combining schemes (with M i.i.d. realizations) with CSIR

- Selection Combining (SC): $\gamma_\Sigma = \max_i \gamma_i$, $P_{\text{out},M} = P_{\text{out}}^M$
- Maximal Ratio Combining (MRC): $\gamma_\Sigma = \sum_i \gamma_i$, $\bar{P}_{s,M} = \bar{P}_{s,1}^M$, can use MGF expressions for $\bar{P}_{s,M}$

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Benefits

- Diversity gain (or diversity order)
- SNR gain (or array gain)

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Benefits

- Diversity gain (or diversity order)
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Can employ MRC and SC at the transmitter also if there is CSIT (transmit diversity)!

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Adaptive systems

Idea

Adapt rate, power, coding, ... to CSIT (fading realization); used all the time in almost all high speed systems

Condition for validity

Channel cannot change too fast! (can be roughly estimated by a markov model and level crossing rates)

Our approach to an achievable adaptive scheme

- Use $P_b = 0.2e^{\frac{-1.5\gamma}{M-1}}$ or $M = 1 + K\gamma$ where $K = \frac{-1.5}{\ln(5P_b)}$
- Use waterfilling ideas to optimize average spectral efficiency ($\log_2(M)$) subject to power constraints thus giving $M(\gamma)$ and $P(\gamma)$
- Use heuristics to take into account discrete M

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System model

Model

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} h_{1,1} & \dots & h_{1,N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r,1} & \dots & h_{N_r,N_t} \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix}}_{\mathbf{x}} + \underbrace{\begin{bmatrix} n_1 \\ \vdots \\ n_{N_r} \end{bmatrix}}_{\mathbf{n}}$$

where $\mathbf{x}$ is what transmitter sends and $\mathbf{y}$ is what receiver sees

Transmit power constraint

$$\mathsf{E}[\mathbf{x}^*\mathbf{x}] = \sum_{i=1}^{N_t} \mathsf{E}[|x_i|^2] \leq \rho$$

Parallel channel decomposition of $\mathbf{H}$

Idea

Use the singular value decomposition (SVD) of channel matrix

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathbf{H}}$$

Parallel channel decomposition

- Transmitter sends $\mathbf{x} = \mathbf{V}\tilde{\mathbf{x}}$ (transmit precoding)
- Receiver obtains $\tilde{\mathbf{y}} = \mathbf{U}^{\mathbf{H}}\mathbf{y}$ (receiver shaping)

$$\tilde{\mathbf{y}} = \boldsymbol{\Sigma}\tilde{\mathbf{x}}$$

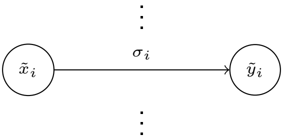

Figure: Equivalent parallel channels (no “crosstalk” or interchannel interference)

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Parallel channel decomposition

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$$\tilde{\mathbf{y}} = \mathbf{\Sigma}\tilde{\mathbf{x}}$$

Note

The number of such channels equals the rank of $\mathbf{H}$

Channel capacity

CSIT and CSIR

$$C = \max_{\mathbf{R_x}:\mathrm{Tr}(\mathbf{R_x})\leq\rho} B\log_2|\mathbf{I}+\mathbf{H}\mathbf{R_x}\mathbf{H^H}| = \max_{\boldsymbol{\rho}:\sum_i \rho_i\leq\rho} B\sum_i \log_2(1+\rho_i\sigma_i^2)$$

Note

Can solve this by waterfilling!

CSIR only

$$C = \max_{\mathbf{R_x}:\mathbf{R_x}=\rho/N_t\mathbf{I_{N_t}}} B\log_2|\mathbf{I}+\mathbf{H}\mathbf{R_x}\mathbf{H^H}| = \max_{\boldsymbol{\rho}:\rho_i=\rho/N_t} \sum_i B\log_2(1+\rho\sigma_i^2/N_t)$$

Beamforming

Idea

Combine multiple antennas to create a single channel with better SNR

Math

Equivalent scalar channel $\tilde{y} = \mathbf{u}^{\mathbf{H}}\mathbf{H}\mathbf{v}\tilde{x} + n$, $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$

Some facts

- SNR optimal if $\mathbf{u}$ and $\mathbf{v}$ are associated with largest singular value of $\mathbf{H}$
- Capacity optimal if largest singular value is much larger than the rest (reason: waterfilling solution interpretation)
- Needs CSIT and CSIR

Diversity multiplexing tradeoff (DMT)

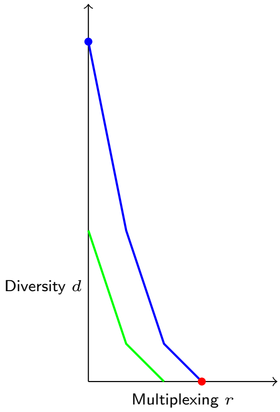

Figure: Blue curve for $N_t = 3, N_r = 3$, green for $N_t = 2, N_r = 2$. Note the piecewise linear nature.

- High SNR concept:
  - Multiplexing gain $r = \lim_{\text{SNR}\to\infty} \frac{R(\text{SNR})}{\log_2(\text{SNR})}$
  - Diversity gain $d = \lim_{\text{SNR}\to\infty} \frac{-\log P_e}{\log \text{SNR}}$
- Valid for complex normal statistics for $H_{i,j}$ (may not be the same curve for different statistics)
- Achievability does not use CSI at transmitter

MIMO receivers (let's say $x_i \in \{-1, +1\}$)

Maximum likelihood (ML), optimal but high complexity

$$\hat{\mathbf{x}} = \text{argmin}_{\mathbf{x} \in \{-1,+1\}^{N_t}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$$

Zero forcing (ZF), suboptimal but linear complexity

$\hat{\mathbf{x}} = \text{sign}(\mathbf{H}^\dagger \mathbf{y})$ where $\mathbf{H}^\dagger = (\mathbf{H}^\mathbf{H}\mathbf{H})^{-1}\mathbf{H}^\mathbf{H}$ if $\mathbf{H}$ is "tall"

Minimum mean squared error (MMSE) (SNR $= 1/\sigma^2$), optimal for gaussian

$$\hat{\mathbf{x}} = \text{sign}((\mathbf{H}^\mathbf{H}\mathbf{H} + \sigma^2\mathbf{I})^{-1}\mathbf{H}^\mathbf{H}\mathbf{y})$$

MIMO receivers continued

Sphere decoders (SD)- Near ML performance

- Use $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 = \|\mathbf{Q}^{\mathbf{H}}\mathbf{y} - \mathbf{R}\mathbf{x}\|^2 = \sum_{i=N_t}^{1}((Q^H y)i - \sum_{j \geq i} R_{i,j} x_j)^2$ to compute $\text{argmin}_{\mathbf{x}:\|\mathbf{y}-\mathbf{H}\mathbf{x}\|<r} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$
- Tuning r trades off complexity versus performance
- Optimal *if and only if* there exists $\mathbf{x}$ within restricted region

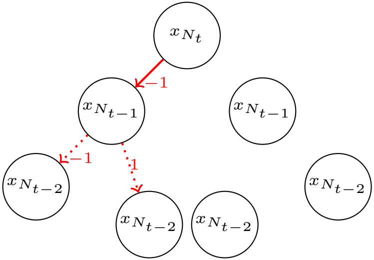

Figure: Tree with 2^{N_t} possible paths (for BPSK). Each node associated with sum of known terms.

Algorithm (depth first search)

- Traverse tree depth first
- Prune branches of tree if accumulated sum at a node is greater than r
- Smaller r leads to lower complexity but possibly higher BER

MIMO receivers continued

Sphere decoders (SD)- Near ML performance

- Use $\|\mathbf{y}-\mathbf{Hx}\|^2 = \|\mathbf{Q^H y}-\mathbf{Rx}\|^2 = \sum_{i=N_t}^{1}((Q^H y)i - \sum_{j\geq i} R_{i,j}x_j)^2$ to compute $\operatorname{argmin}_{\mathbf{x}:\|\mathbf{y}-\mathbf{Hx}\|<r} \|\mathbf{y}-\mathbf{Hx}\|^2$
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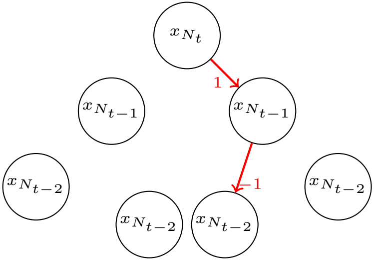

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Multicarrier modulation

Idea

Divide large bandwidth into smaller chunks and use narrowband signals

Advantages

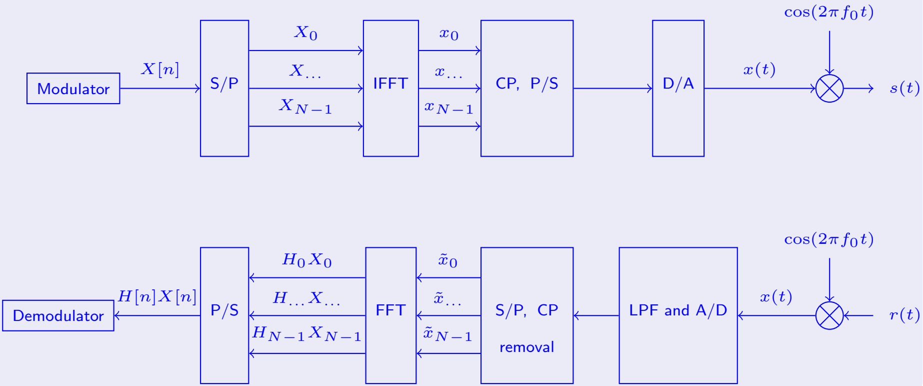
- Takes care of intersymbol interference (ISI)
- Multiplexing subcarriers (OFDMA)
- Signal processing can be extremely efficient

Multicarrier modulation

Idea

Divide large bandwidth into smaller chunks and use narrowband signals

OFDM block diagram



Multicarrier modulation

Idea

Divide large bandwidth into smaller chunks and use narrowband signals

Fineprint

- Use FFT/IFFT for frequency time interconversion ($\Theta(N \log N)$ complexity)
- Use cyclic prefix to simulate circular convolution from linear convolution with finite impulse response
- Subchannels may be used for diversity, multiplexing, depending on how correlated they are

Spread spectrum

Idea

Spread a narrowband signal over a wider band

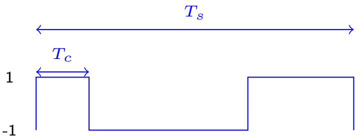
Some common methods

- FHSS
- DSSS

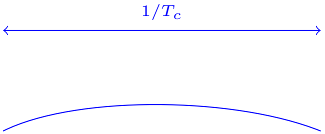
DSSS idea

- At transmitter: Use a spreading code (also known as chip sequence) $s_c(t)$ of bandwidth $B = 1/T_c$ (sometimes called *chip rate*) with which to multiply narrowband signal $g(t)$ of duration $T_s = 1/B_s$
- At receiver: Take the integral of $r(t)s_c(t)$ over time T_s
- Processing gain $\triangleq \frac{B}{B_s}$

Some properties of the spreading code $s_c(t)$



$s_c(t)$ in the time domain



$S_c(f)$ frequency domain

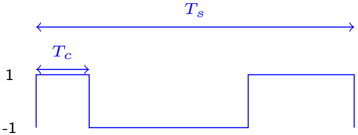
- At transmitter: Send $s(t) = g(t)s_c(t)$, $g(t)$ narrowband, duration T_s
- At receiver: Compute $\frac{1}{T_s}\int_0^{T_s} s(t)s_c(t)dt$

Narrowband interference rejection

- Received signal $r(t) = s(t) + i(t) = s_c(t)g(t) + i(t)$
- Receiver processing $\frac{1}{T_s}\int_{t=0}^{T_s} r(t)s_c(t)dt \approx \bar{g} + \frac{1}{T_s}\int_{t=0}^{T_s} i(t)s_c(t)dt$

Here $\bar{g} = \frac{1}{T_s}\int_{t=0}^{T_s} g(t)dt$

Some properties of the spreading code $s_c(t)$



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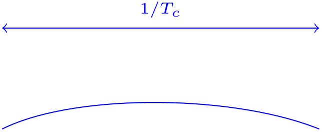

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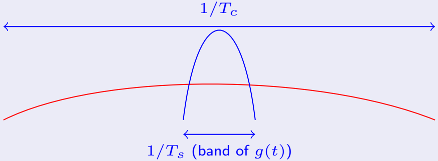
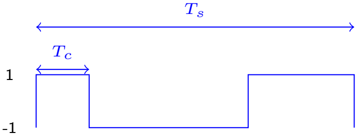
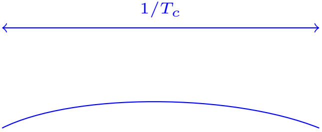


Frequency Domain of $i(t)s_c(t)$

Some properties of the spreading code $s_c(t)$



$s_c(t)$ in the time domain



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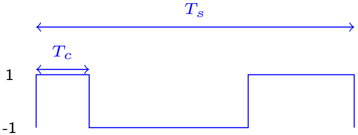
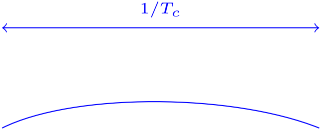
Narrowband interference rejection

Interference energy in the band of $g(t)$ is reduced by approximately T_s/T_c!

Some properties of the spreading code $s_c(t)$



$s_c(t)$ in the time domain



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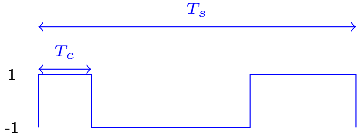
- At receiver: Compute $\frac{1}{T_s}\int_0^{T_s} s(t)s_c(t)dt$

Multipath(ISI) rejection

- Received signal $r(t) = s_c(t) + \alpha s_c(t-\tau)$
- Receiver signal processing $\frac{1}{T_s}\int_0^{T_s} r(t)s_c(t)dt = \bar{g}\rho(0) + \frac{\alpha\bar{g}}{T_s}\rho(\tau)$

Some properties of the spreading code $s_c(t)$



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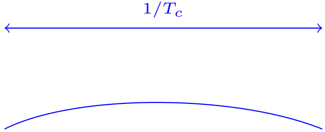

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Multipath(ISI) rejection

- The support of $\rho(\tau)$ needs to be concentrated around $\tau = 0$ for good multipath rejection
- Larger support, on the other hand, good for synchronization/ acquisition of phase (why ?)

Rake receivers

Idea

Using spreading codes with good ISI rejection, we can distinguish different multipath components!

Some facts

- RAKE receiver simply gathers energy from different multipath components with different delays
- Different branches of the RAKE receiver synched to a different delay component
- Can be combined using diversity combining techniques (MRC/SC, etc.)

Outline

1. Review
   - Channel models
   - Performance analysis
   - Combating fading using diversity
   - Combating fading using adaptive modulation and power
   - Point to point MIMO systems
   - Combating multipath/ISI/small B_c
2. Sample finals/discussion

MIMO

Consider the following channel gain matrix:

$$\mathbf{H} = \begin{bmatrix} 0.1 & 0.3 & 0.7 \\ 0.5 & 0.4 & 0.1 \\ 0.2 & 0.6 & 0.8 \end{bmatrix}$$

$$= \begin{bmatrix} -0.555 & 0.3764 & -0.7418 \\ -0.333 & -0.9176 & -0.2158 \\ -0.7619 & 0.1278 & 0.6349 \end{bmatrix} \begin{bmatrix} 1.333 & 0 & 0 \\ 0 & 0.5129 & 0 \\ 0 & 0 & 0.0965 \end{bmatrix}$$

$$* \begin{bmatrix} -0.2811 & -0.7713 & -0.5710 \\ -0.5679 & -0.3459 & 0.7469 \\ -0.7736 & 0.5342 & -0.3408 \end{bmatrix}$$

Assume the system bandwidthis $B = 1$ MHz, the noise power is 0 dBm and perfect CSI at TX and RX. You may use the approximation BER $\approx 0.2e^{-1.5\gamma/(M-1)}$.

MIMO

What are the transmit precoding and receiver shaping matrices associated with beamforming (1 spatial dimension) and 2D precoding (2 spatial streams)?

MIMO

Find the capacity when the transmit power is 10 dBm. Can the transmit power affect the optimal number of spatial streams?

MIMO

MIMO

Find the data rate that can be achieved with optimal adaptive modulation across spatial dimensions for a total transmit power of 20 dBm, assuming unconstrained MQAM and BER target of 10^{-4}.

MIMO

MIMO

For a transmit power of 20 dBm and MQAM constellations constrained to no transmission, BPSK or $M = 2^k, k = 2, 3, 4, \ldots$, a target BER of 10^{-4} and power divided equally among all spatial streams, find the total data rate associated with all data streams under beamforming, 2D precoding and using all spatial streams.

MIMO

MIMO

For 16QAM modulation and a 20 dBm transmit power equally divided across all spatial streams, find the BER for each stream under beamforming, 2D precoding and spatial multiplexing.

OFDM

Consider an OFDM system with N subchannels and flat fading on each subchannel. The system uses an appropriate length cyclic prefix to remove ISI between FFT blocks so the ith subchannel can be represented as $Y[i] = H[i] \odot X[i] + N[i]$ where $H[i]$ is the fading assocated with the ith subchannel.

OFDM

Suppose the delay spread is 10 μs. If the total channel bandwidth is 10 MHz and the OFDM system has an FFT size that must be a power of 2, what size FFT ensures flat fading on each subchannel?

OFDM

Assume an OFDM system with 8 subchanels each with a bandwidth of 100kHz. With 400mW transmitted on each subchannel, the received SNR is $\gamma_i = 400/i$ (linear units). Given a total transmit power of $P = 400$ mW total across subcarriers, what is the capacity of your system when transmit power is constant across subcarriers?

OFDM

What is the capacity when power is adapted so there is constant receive SNR on each subcarrier?